Adam Drozdek

In the beginning was the *apeiron*

Infinity in Greek philosophy

Klassische Philologie

Palingenesia – Band 94

Franz Steiner Verlag

PALINGENESIA

Schriftenreihe für
Klassische Altertumswissenschaft

Begründet von Rudolf Stark
nach Otto Lendle und Peter Steinmetz
herausgegeben von

SEVERIN KOSTER

Band 94

Adam Drozdek

In the beginning was the *apeiron*

Infinity in Greek philosophy

Franz Steiner Verlag Stuttgart 2008

A11
.D769
2008

Bibliographische Information der Deutschen Bibliothek
Die Deutsche Bibliothek verzeichnet diese Publikation in der Deutschen Nationalbibliographie; detaillierte bibliographische Daten sind im Internet über <http://dnb.ddb.de> abrufbar.

ISBN 978-3-515-09258-6

Jede Verwertung des Werkes außerhalb der Grenzen des Urheberrechtsgesetzes ist unzulässig und strafbar. Dies gilt insbesondere für Übersetzung, Nachdruck, Mikroverfilmung oder vergleichbare Verfahren sowie für die Speicherung in Datenverarbeitungsanlagen.
Gedruckt auf säurefreiem, alterungsbeständigem Papier.
© 2008 Franz Steiner Verlag, Stuttgart
Druck: Printservice Decker & Bokor, München
Printed in Germany

TABLE OF CONTENTS

Preface 7

1 Anaximander 9

- 1.1 *Apeiron* 9
- 1.2 Infinity of worlds 13
- 1.3 Origin 16
- 1.4 Naming the *arche* 18
- 1.5 Anaximenes 19

2 The Pythagoreans 21

- 2.1 Harmonious universe 21
- 2.2 Limiters and unlimiteds 22
- 2.3 Numbers 25

3 Xenophanes 29

- 3.1 Infinite world 29
- 3.2 Infinite God 31

4 Heraclitus 35

- 4.1 Eternity 35

5 Parmenides and Melissus 40

- 5.1 Unlimited and infinite 40
- 5.2 Sphere 42
- 5.3 Philosophical environment 43
- 5.4 Incorporeality of Being 46

6 Zeno's paradoxes 48

- 6.1 The dichotomy 48
- 6.2 The Achilles 50
- 6.3 The flying arrow 50
- 6.4 The stadium 52
- 6.5 Plurality 54
- 6.6 Infinity 55

7 Empedocles 57

- 7.1 Eternal cycles 57
- 7.2 The sphere 58

8 Anaxagoras 61

- 8.1 Physics 61
- 8.2 The everything in everything principle 66
- 8.3 The infinitely large 74

9 The atomists 76
9.1 Atoms and the void 76
9.2 Infinity of worlds 79
9.3 Infinite divisibility 81
10 Plato 85
10.1 Finite world 85
10.2 Numbers 88
10.3 Infinity in the *Philebus* 91
10.4 The *Philebus* and the *Timaeus* 95
10.5 Time and eternity 96
10.6 Knowing the infinite 98
11 Aristotle 101
11.1 Defining infinity 101
11.2 Aristotle's razor 102
11.3 Infinity by addition 103
11.4 Continuum 106
11.5 Causality 110
11.6 Eternity 111
11.7 The infinite and the finite 114
12 The Old Academy 116
12.1 The unwritten doctrines 116
12.2 Speusippus 117
12.3 Xenocrates 121
13 The Early Lyceum 126
13.1 Theophrastus 126
13.2 Eudemus 129
13.3 Strato 130
14 Epicurus 133
14.1 Atoms and the void 133
14.2 Infinite worlds 139
14.3 Knowledge of the infinite 140
15 The Early Stoa 142
15.1 The void 142
15.2 Time 145
15.3 Continuum 149
15.4 *Lekta* 151
15.5 The finite and the infinite 154
16 Infinity in Greek mathematics 155
16.1 Nonnegative integers 155
16.2 Incommensurability 157
16.3 Quadrature of the circle 158
16.4 Euclid and Archimedes 162
Index of names 166
Index of ancient sources 170

PREFACE

The beginning of Greek philosophy is marked by the realization about the status and importance of infinity, as testified by Anaximander's philosophical system. Since Anaximander marks the transition from mythology to philosophy, we can agree with the statement that "in 'infinity' thought gained the first realization of itself. It was a battle cry with which speculation entered the level directed against banal 'truths' of sensory perception."[1] This battle was continued all throughout Antiquity, and, directly or indirectly, all philosophical discussions after Anaximander were somehow related to the concept of infinity in its various aspects, including the temporal and spatial aspects.

There are many different dimensions of infinity in general and in Greek philosophy in particular. One dimension is theological: the question of God's infinity and eternity, the problem of divine infinite knowledge and power and the like, and these problems were investigated in my book *Greek philosophers as theologians* (Ashgate 2007). Another dimension of infinity pertains to ontology and physics: the problem of the infinite size of the world and of its eternal existence, the problem of the infinity of worlds, of the infinite divisibility of matter, of the infinity of attributes or attribute modes (e.g., infinity of atom shapes), the problem of the infinity of nonphysical entities such as mathematical constructs. These are the problems investigated in this book. The third aspect of infinity is anthropological, in particular the problem of immortality and eschatology, and is a problem that deserves separate investigation.

Infinity entered the Greek intellectual history very forcefully with Anaximander's divine *Apeiron* and is a foundational concept – whether explicitly, or, more frequently, implicitly – of all Greek philosophical systems. In the sixth and the fifth century philosophical systems, infinity was unproblematic. It was taken for granted as something both obvious and also indispensable. This can be seen, for instance, in the way the existence of the world was perceived: the universe was characterized by infinite existence. It may be the same universe that is eternal; many universes that exist concurrently; many universes that are generated and dissolved, but there was always a world; the world that was molded from eternally existing substance by an eternally existing being or simply generated by an underlying substance (atoms) configuring itself according to eternally existing natural laws. The world, in a way, is eternal or the substrate from which it arose always existed. In that sense, infinity precedes the finite, the infinite has to be – and is – assumed before speaking about the finite: a finite world whether in respect to its duration or to its size.

---

[1] Albert Görland, *Aristoteles und die Mathematik*, 1899 [reprint, Frankfurt/M.: Minerva 1985], 157.

That infinity as naturally assumed is far from obvious was brought to realization by the Zeno's famous paradoxes. And yet infinity was unavoidably the basis of the solution proposed to Zeno's challenge; the proposed solutions included an eternally static world of Parmenides, a world filled with an infinity of eternally existing atoms, or – most impressively – Anaxagoras' world which was structured and infinitely self-repeating at each level of magnitude. Only with Aristotle did infinity truly become the object of philosophical reflection and that also marked, in a way, a low point of infinity in Greek thought, when infinity was explicitly pronounced a not quite desirable or welcome attribute. In spite of Aristotle's reservations, infinity figures prominently in the philosophical systems of Epicurus and the Stoics. Infinity was also used in mathematics as a fundamental and obvious assumption.

This book presents the views of philosophers on the issue of infinity from the first Presocratics to those during the early Hellenistic period. The problem of infinity in Greek mathematics is also briefly presented. Greek intellectual history clearly shows that the Greeks had none of the *horror infiniti* that sometimes is ascribed to them. "The reticence of modern interpreters on the subject of the concept of infinity in Ancient Greece seems hard to justify. The idea of the extension without limit in length, width, depth, or, generally, in volume, does not seem to be so subtle or sophisticated that it could not be envisioned by a thinker of the sixth century."[2] Infinity was woven almost effortlessly into their philosophical systems and into mathematical theories. Zeno's concerns and Aristotle's reservations did not prevent infinity's prominent position in philosophy and science. Far from it, as the example of Anaxagoras shows; it only led to elevating the use of infinity to new levels.

ACKNOWLEDGMENTS

I would like to thank the editors of *Hermes* for their permission to include parts of "Eleatic Being: finite or infinite?" in chapter 5; "Anaxagoras and the everything in everything principle" in chapter 8, and "Infinity in Chrysippus" in chapter 15; the editors of *Diálogos* for their permission to include modified and extended text of "Aristotle's razor" as chapter 11; and the editors of *Eos* for their permission to include "Infinity in Plato" as chapter 10.

---

[2] Alexandre P.D. Mourelatos, La terre et les étoiles dans la cosmologie de Xenophane, in A. Laks, C. Louguet (eds.), *Qu'est-ce que la philosophie présocratique?*, Villeneuve d'Ascq: Presses Universitaires du Septentrion 2002, 335-336.

1 ANAXIMANDER

Infinity enters forcefully into the Greek philosophical scene from the very inception of philosophical thought in Greece. In fact, it is the reflection on infinity which made this philosophy possible, which was the prime mover of intellectual activity leading to philosophy. The roots of this thought are theological in nature. When philosophers started to think about the nature of the divine in order to rectify it from popular anthropomorphic accretions and free it from behavior the epic poems allowed gods to have, the infinity came as one attribute constituting the divine. Thus, it is fitting that at the dawn of Greek philosophical reflection infinity becomes the *arche*, the principle and the beginning of what exists. Truly, at the beginning was the *Apeiron*.

1.1 *APEIRON*

In the words of Theophrastus as preserved by Simplicius, "of those who hold that [principle] is one, moving, and infinite, Anaximander, son of Praxiades, a Milesian, a successor and pupil of Thales, said that the *Apeiron* is the principle and element of existing things; [he was] the first to introduce the name 'principle.' He says that it is neither water nor any other of the so-called elements, but some other *Apeiron* nature from which come into being all the heavens and the worlds in them; [things] from which is the source of existing things, into these they are destroyed in accordance with necessity; for they give justice and retribution to each other for their injustice according to the order of time, as he says in rather poetic words. It is clear that when observing the change of the four elements into one another he was unwilling to make any one of them the substrate but [chose] something else apart from them. He explains coming into being (*genesis*) not by alteration of the element but by separating off of the opposites by the eternal motion" (*In Phys.* 24.13-25 = 12A9 + B1). Also, "he said that the principle of all existing things is some nature [under the heading] of the *Apeiron* from which the heavens are generated and the cosmos in them; it is eternal and unaging and it envelops all worlds (*kosmoi*). He speaks about time as determinate by generation and existence and destruction are" (A11). The *Apeiron* is the source and the end of all things, principle of the universe that governs everything; it is also "divine because it is immortal and indestructible" (A15).

The nature of the *Apeiron* has nearly always been controversial. Already in antiquity, at least four different views concerning its nature can be distinguished: the *Apeiron* is 1. something indeterminate; 2. some intermediate substance; 3. a mixture, and 4. potentiality.[1]

---

[1] Friedrich Lütze, *Über das Apeiron Anaximanders*, Leipzig: Julius Klinkhardt 1878, 5.

Concerning the indeterminate nature of the *Apeiron*, Theophrastus stated that if Anaxagoras' mixture were considered to be "one nature, indeterminate in respect to essence and magnitude," as Anaxagoras appears to have believed, then Anaxagoras' material principle would be just like Anaximander's *Apeiron* (A9a; 59A41). The *Apeiron* is thus an indeterminate nature (φύσις ἀόριστος), indeterminate in two respects: its essence and its magnitude.[2] Indeterminacy in respect to essence means that we cannot quite state what it is, because we cannot specify its properties, in contrast, for instance, to Anaximenes whose principle is determinate, namely air (13A5). Also, the *Apeiron* expands infinitely in space; its limits cannot be determined not because they are unknown or fuzzy, but because there are no limits at all.

It is also claimed that the *Apeiron* is an intermediate substance. Aristotle investigated whether there are many elements or one. He said that "some assume only one, which, for some, is water, for some, air, for some, fire, and for some, something finer than water and denser than air and which, being infinite, envelops, as they say, all the heavens" (*De caelo* 303b9-13). Is this an allusion to Anaximander? If it were, then there would be a conflict between the indeterminate nature of the *Apeiron* mentioned by Theophrastus and the determinate level of fineness of the intermediate element to which Aristotle alluded. Therefore, either an unspecified philosopher who is unknown to us is meant by Aristotle,[3] or Aristotle did not consider the *Apeiron* to be indeterminate. There is a similar problem with Aristotle's statement that "all the physicists suggested for the *Apeiron* some other nature [other than the nature proposed by Pythagoreans and Plato, namely one] of the so-called elements, such as water, air, and their intermediate (τὸ μεταξύ)" (*Phys.* 203a16-18). If the word "all" is taken literally, Anaximander is also included among the physicists whose views Aristotle presented. This leads to the already mentioned inconsistency between the determinate nature of the water-air intermediate substance and indeterminacy of the *Apeiron* presented by Theophrastus. Instead, maybe "all the physicists" means "all the physicists who can be taken seriously" or "all reasonable physicists." From Aristotle's point of view, some tenets of Anaximander's system are absurd (ἄτοπον, *Phys.* 208a3), and thus Anaximander's views should not be considered in all seriousness.[4] Aristotle's rather decisive statement, however, clearly counts Anaximander among those who did not believe in an intermediate element or, in fact, any element. Aristotle divided philosophers into "those who make the body-substrate one – either one of the three [water, air, or fire] or something else which is denser than fire and finer than air" and those who assert that "the contraries contained in the one emerge from it by segregation, as Anaximander says" (*Phys.* 187a12-15,20-21 = 12A16). Clearly, the first group can also include philosophers who claimed the

---

[2] Therefore, Theophrastus used the world *aoristos*, not *apeiros*, since the latter is ill-suited to convey the indeterminacy in respect to essence, as observed by Marcel Conche, *Anaximandre: fragments et témoignages*, Paris: Presses Universitaires de France 1991, 74.

[3] W.D. Ross proposed a disciple of Anaximenes as a possible candidate, Aristotle, *Metaphysics*, Oxford: The Clarendon Press 1924, v. 1, 178.

[4] Conche, *op. cit.*, 91.

existence of an element that is intermediate between any pair of original elements, in particular, the water-air intermediary mentioned in the previous fragment just as this group would also include a philosopher claiming earth as the primal element had there been any (and none is known, *Met.* 989a5). Also, it would be much more meaningful if Anaximander came after Anaximenes so that his principle could be something between Thales' water and Anaximenes' air. As it stands, the theory of an intermediate element can be introduced only in hindsight, after the two elements were introduced, not to mention other elements that become entrenched in Greek thought with Empedocles. Also, the fact that in one place Aristotle spoke about the water-air intermediate and in another about the fire-air intermediate – and in yet another place about the water-fire intermediate (*Phys.* 189b3) – can be taken to mean that Aristotle's memory failed him in at least two places when he allegedly alluded to Anaximander or that these intermediates were mentioned as hypothetical possibilities without any physicist making claims about their existence.

According to the third interpretation, the *Apeiron* is a mixture. "Everything originates from being if at least potential being is understood, not actual being: and such is the one of Anaxagoras ... and the mixture (τὸ μῖγμα) of Empedocles and of Anaximander" (*Met.* 1069b19-22 = 59A61). The text is corrected, for example, by interchanging the names of Anaxagoras and Anaximander,[5] or by inserting "*apeiron*" after Anaximander's name.[6] However, if Anaximander's mixture is understood as being something in which everything exists potentially and is separated off from it, directly or indirectly, then this mixture of potentialities would correspond to the *Apeiron* since things, in particular, the opposites, "were no more pre-existent in the *ἄπειρον* than children pre-exist in the body of their parents before conception."[7] Empedocles would later describe an actual mixture, a mixture as actual being, since it is a mixture of the four elements more or less organized depending on the stage of the cosmic process.

The fourth interpretation of the *Apeiron* sees in this principle a counterpart of potentiality, of matter. Aristotle wrote that "when distinguishing four kinds of causes, it is clear that the *apeiron* is a cause as matter ... and also all other [physicists] clearly used the *apeiron* as matter; that is why it is absurd to make it what envelops but not what is enveloped" (*Phys*. 207b34-208a4 = 12A14). The *Apeiron* is indeterminate and, generally, the indeterminate cannot envelop the determinate (207a20-32). Thus, whatever is infinite can only be enveloped (207a25). In Aristotle's system, the finite has a hold on the infinite. This is clearly incompatible with Anaximander, who simply reversed the roles: the infinite is primary, the finite secondary, and the latter is determined only in the framework of the former, while the finite is determined by the governing ability on the *Apeiron*. Aristotle's argument is valuable when the *Apeiron* is identified with matter. Matter as understood by Aristotle is indeterminate (*Met*. 1029a20-21), uncreated, and indestructible (*Phys*. 192a28), just like the *Apeiron*. However, matter does not exist in separation and, as such, it is not a substance (*Met*. 1029a26-30). Also, matter is passive (*De gen. et corr*. 335b29-31). The *Apeiron*, on the other hand, exists as a separate substance which is active, itself being the prime cause of all other beings.[8] Therefore, there is nothing absurd in Anaximander's position if his understanding of the *Apeiron* is not equated with Aristotle's view of infinity.

---

[5] Lütze, *op. cit.*, 54, who followed Schleiermacher.

[6] G. Calogero, see G.S. Kirk, J.E. Raven, M. Schofield, *The Presocratic philosophers*, Cambridge: Cambridge University Press 1995, 130 note 1.

[7] Charles H. Kahn, *Anaximander and the origins of Greek cosmology*, New York: Columbia University Press 1960, 236. This point should not be pressed too much since, as asked rhetorically by Büsgen, "isn't in this sense any principle of matter a potential μῖγμα, and thus it can be stated, for example, about Anaximenes' air that it contains δυνάμει everything, since everything develops from it?" [Ferdinand] Büsgen, Über das *Apeiron* Anaximanders, in *Programm, mit welchem zur öffentlichen Prüfung und Schlussfeierlichkeit des Königlichen Gymnasiums zu Wiesbaden auf den 8. und 9. April 1867 geziemend einladet der Direktor des Gymnasiums Dr. Karl Schwartz*, Wiesbaden: Schellenberg 1867, 3. It merits mentioning that Büsgen himself advocated the understanding of the *apeiron* as mixture. A suggestion is made that "very probably he [Anaximander] did think of the Infinite as containing particles of contrary things," John B. McDiarmid, Theophrastus on the Presocratic causes, *Harvard Studies in Classical Philology* 61 (1953), 101.

It appears that the first interpretation is the most convincing. The *Apeiron* is an infinite and eternal substance; i.e., it is infinite spatially and temporally, it is indestructible, the ultimate source of all being, whose essence is indeterminate. The latter specification creates some tension: if the *Apeiron* is indeterminate, how can we say it is infinite and indestructible? These two attributes are privative and thus somewhat compatible with indeterminacy. However, the *Apeiron*, having no beginning or end, becomes eternal, and eternity is not a privative attribute. Moreover, the *Apeiron* governs everything which can be meaningfully done when it is assumed to be a rational entity.[9] However, maybe eternity and rationality, although cannot constitute the indeterminate essence of the *Apeiron*, can be attributed to its manifestations. This would correspond to the apophatic theology of Orthodoxy according to which the essence of God is not known and is unknowable,[10] but God's goodness, wisdom, eternity, etc. can be attributed to God's energies, i.e., to God's impact on His creation.[11]

---

[8] Conche, *op. cit.*, 87-90.

[9] Governing "can be predicated only about an active person, who like a helmsman sits at the helm of the cosmic ship," Olof Gigon, Die Theologie der Vorsokratiker, in *La notion du divin depuis Homère jusqu'à Platon*, Genève: Vandoeuvres 1954, 136; cf. Michael C. Stokes, Anaximander's argument, *Canadian Journal of Philosophy*, Supp. 2 (1976), 17.

[10] Just as Aristotle's matter is in itself unknowable (*Met.* 1036a8-9) and can be characterized neither positively nor negatively (1029a24-26).

[11] Burch saw in Anaxagoras an anticipation of negative theology of Dionysus the Areopagite, George B. Burch, Anaximander, the first metaphysician, *Review of Metaphysics* 3 (1949-50), 143-144.

1.2 INFINITY OF WORLDS

Theophrastus' account given by Simplicius in A9, considered the closest to the source,[12] mentions "all the heavens and the worlds in them"[13] originating in the *Apeiron*, but it does not commit itself to the infinity of worlds nor to the way the worlds exist, either successively or concurrently. Simplicius also stated somewhat hesitatingly that Anaximander "was first to posit the *apeiron* to be able to put it plentifully to use in generation; and, as it seems, he posited infinite worlds and that each of the worlds is generated from such an infinite element" (*In De caelo* 615.15 = A17). He also said that "having posited a principle that is infinite in size, Anaximander seems to produce from it worlds that are infinite in number" (202.14). Only once did Simplicius firmly report Anaximander's opinion about the infinity of worlds; he said that followers of Anaximander (and, by extension, Anaximander himself) believed in the existence of infinite worlds "that are generated and destroyed to infinity and some are always being generated and others are destroyed; and they say that motion is everlasting" (*In Phys.* 1121.5-9 = A17). Simplicius, the most reliable source about Anaximander, appears uncertain about whether or not Anaximander maintained the existence of infinite worlds. Other sources express no hesitation in that respect; for example, we read that according to Anaximander, "the infinite heavens are gods" (A17) and "infinite worlds are generated and are destroyed back into that from which they are generated" (A14).

There is, however, a question about how this infinity should be understood. Is this an infinity of successive universes, when creation of one follows the decline of the other, or is it an infinity of universes existing at the same time? The ancient testimonies allow for both interpretations.[14] One source states that the heavens are separated off from the *Apeiron*, and, "in general, all the worlds that are infinite." Destruction and generation have taken place since infinite time, and come in a cycle (A10). The testimony undoubtedly points to an infinity of successive worlds, but it does not exclude the possibility that in concurrent worlds such cycles can also be present. Similarly, Anaximander believed that the principles of particular things are infinite,[15] and they "generate innumerable worlds and whatever things are born in them; and he thought that these worlds are dissolved at one time and generated again at another time, each for as long as it can last" (A17). This testimony favors the infinity of successive worlds, but it is not incompatible with the infinity of coexisting worlds. Also, the Hippolytus' phrase that the *Apeiron* "envelops all worlds" (*Ref.* 1.6.1 = A11) is much more meaningful when it is understood that the *Apeiron* surrounds them at the same time rather than in succession.[16]

---

[12] Richard McKirahan, Anaximander's infinite worlds, in A. Preus (ed.), *Essays in ancient Greek philosophy VI: before Plato*, Albany: State University of New York Press 2001, 53, 62.

[13] The phrase is claimed to have come from Anaximander himself since Theophrastus, a peripatetic, would hardly used it, Kahn, *op. cit.*, 49, although the view is contested by McKirahan, *op. cit.*, 55. In this phrase, "all the *ouranoi* and the *kosmoi* in them," *ouranos* should be taken in its natural meaning as the system of celestial bodies, and *kosmos* as the world or the world system (not just part of the world), and the phrase itself does not force us to see multiple *kosmoi* in one *ouranos*, but it may simply mean that there is one *kosmos* is one *ouranos*, i.e., one system of celestial bodies is characterized by a particular world-system or world-order, cf. McKirahan, *op. cit.*, 56-57.

[14] There is also an interpretation that sees only one world which was created but it has no end, G.S. Kirk, Some problems in Anaximander, *Classical Quarterly* 5 (1955), 28-32; Kahn, *op. cit.*, 1960, 47-53.

[15] Augustine confounded here the views of Anaximander and Anaximenes, Kahn, *op. cit.*, 42, Paul Seligman, *The apeiron of Anaximander*, London: The Athlone Press 1962, 43, Conche, *op. cit.*, 99.

Some testimonies can be interpreted either way. According to one of them, Anaximander said that gods are born "at long intervals" and perish, and that "these are innumerable worlds" (A17). This fragment can be used in interpreting the successive infinity of worlds and coexistent infinity depending on the interpretation of "intervals," which can be understood in temporal or in spatial sense. Similarly with the statement that according to "Anaximander, Anaximenes, Xenophanes, Archelaus, Diogenes, Leucippus, Democritus, and Epicurus, there are infinite worlds in the *apeiron* in every direction" (A17), which can be interpreted either way depending on how "direction" is understood, although spatial meaning and thus coexistent interpretation is more natural.[17] However, it would be difficult to see anything else but the infinity of coexisting worlds in the statement that "of those who claimed that there are infinite worlds, Anaximander held that they are equidistant from one another" (A17). In sum, the sources are noncommittal, or they allow for successive and concurrent infinity, or they favor the latter. It appears that the successive infinity of worlds would be much less controversial for Anaximander's contemporaries and for his doxographers than the simultaneous existence of an infinity of worlds, or even a multiplicity of worlds. That is why this aspect of Anaximander's thought appears in sources so frequently. The process of generation and destruction can be observed in everyday life; projecting it onto a cosmic scale does not require a great leap of abstract thought. However, envisioning a concurrent multiplicity of worlds is an entirely different matter.

The possibility for a truly infinite number of worlds is reinforced by the argument that the *Apeiron* was introduced as the principle "to be able to put it plentifully to use in generation" of the worlds (A17), or "so that the existing process of generation does not fail" (A14). If the worlds had been finite in number and size, no infinite substance should have been necessary to generate them. Moreover, if the worlds were created successively, the material left from the destruction of the current world could be used to generate the next world. Thus, no infinite substance would be needed to ensure an infinity of successive worlds,[18] unless it is claimed that the material from the current world could not be recycled. However, everything comes from the *Apeiron* and eventually turns back to it, thus the end product of the destruction of each world is integrated with the *Apeiron* from which new worlds are generated. Therefore, the argument of plentifulness

---

[16] Cf. Conche, *op. cit.*, 119-120.

[17] Cf. Conche, *op. cit.*, 109-110.

[18] As phrased by Aristotle, "the destruction of one thing can be the origin of another, the universe being limited" (*Phys.* 208a9-11).

used in the generation of worlds is more cogent when it is assumed that an infinity of worlds exists concurrently. However, the argument itself seems to belong to doxographers rather than to Anaximander. When taken at face value, the argument says that Anaximander first allowed for an infinity of worlds in his cosmic system, and then sought for a principle which would make it possible. However, it seems that it was the other way around. Anaximander first established the *Apeiron* as the principle of being, and then allowed for an infinity of worlds. Why an infinity of worlds? According to Anaximander, our universe has the earth in its center, with the sun merely 27 earth diagonals apart (and fixed stars even closer than the sun, A11, the view which later can be found in Leucippus, 67A1.33). Maybe such a tiny universe appeared to him to be overwhelmingly imbalanced size-wise in comparison with the enveloping *Apeiron* that is infinitely extended in each direction. It seems that a cosmic balance is restored not only when the divine *apeironic* substance is infinite, but when the ordinary physical substances stemming from it are infinite as well. Also, theologically, does one such small universe do justice to the divine *Apeiron*, a power infinite in space and duration? An infinite number of universes does it much better – infinitely better.

The possibility that an infinity of worlds may coexist raises an interesting problem of different levels of infinity. How much substance is needed to generate them? An adaptation of Zeno's bisection argument could be used to argue that an infinity of worlds can be created with a finite amount of substance: world 1 is created with amount x, world 2 with amount $x/2$, world 3 with amount $x/4$, generally, world i with amount $x/2^{i-1}$, so that $2x$ units of substrate would be sufficient for all the worlds. However, it is rather unlikely that Anaximander would like to follow this argument. The worlds would get smaller and smaller, until they would become too small for a world (although, in another form, minuscule sizes of things will later appear in Anaxagoras). It seems that for Anaximander, an infinity of worlds required infinite amount of substance separated off from the *Apeiron*.[19] What is left after this separation? Would this process of separation-off result in exhausting the *Apeiron*? This is a real possibility. If so, the *Apeiron* would exist in an unoriginal form as the matter of different universes. That, however, would undermine its purity – the worlds and the things in them would be unfavorably modified, damaged, or destroyed – so that in such modified form, the *Apeiron* would be prone to errors, and its soundness would be severely compromised. Could such an *Apeiron* play the role of a steersman of all the universes? Could it be called divine? Hardly. It seems, then, that the process of separation-off of worlds does not exhaust the *Apeiron*. Even in the presence of an infinity of worlds, the *Apeiron* in pure form would still be infinite. Analogically, the number of integers is infinite, and there is still an infinity of odd integers left after separating even numbers from all integers. The existence of an infinity of worlds does not have to compromise infinity, and thereby, divinity, of the *Apeiron*.

---

[19] The physical process of separation is tantamount to theological process of emanation which means that at the beginning was the *apeiron*, and nothing else, unlike, for example in Plato's system where the Demiurge molded preexisting matter to create, or rather form, the world, cf. Burch, *op. cit.*, 145.

Aristotle reported that Anaximander believed the earth to be at rest because of its similarity (*homoioteta*); "for it is no more fitting for what is established at the center and is equally (*homoios*) related to the extremes to move up rather than down or to the sides; and it is impossible for it to make a move simultaneously in opposite directions; therefore, it is at rest by necessity" (*De caelo* 295b10-16 = A26). The cylindrical earth remains motionless in the middle of the world because of the symmetry of distances of different points of the earth from the heaven in which it is enclosed. The top of the earth is removed from the top of the heavens by the same distance as earth's bottom from heaven's south side; similarly, each point on the circumference of the earth is removed from a point of the heavens by the same distance as the point on the opposite side of the diagonal is removed from another point of heaven. This can best be assured when heaven is spherical, and it appears that Anaximander believed in a spherical shape of heaven. This idea is supported by a cosmogonic report stating that at one point in the world formation, a sphere of flame was separated off from the *Apeiron* (A10). Other reports say that there are solid rings in which celestial bodies are apertures like the rims of chariot wheels (A11, A21, A22) and even that celestial bodies are carried by circles and spheres (A18). The earth is thus enclosed in a spherical heaven, and outside the heaven there is the *Apeiron* in its unvarnished substantial form. What about other worlds? Are they like ours? Possibly, but Anaximander's cosmic system does not require the universe to be populated by identical celestial spheres with identical celestial bodies inside, and with identical cylindrical earths in the middle. It seems that Anaximander would permit an infinity of spherical heavens of different diameters with different sets of celestial bodies inside, and with differently shaped planets in the middle of each sphere.[20] The space between different celestial spheres would be filled with the *Apeiron*, which could be touched and even sensed by someone reaching the extremity of a particular heaven. Celestial spheres with worlds inside them are in constant flux, and new heavens are brought into existence by the inscrutable will of the divine *Apeiron*; existing heavens perish by being dissolved into primal *apeironic* substrate.

1.3 ORIGIN

What is the origin of the grand idea of an infinite principle? The primary source of the idea of the *Apeiron* seems to be religious in nature. Greek culture in Anaximander's time was saturated with religion. Religion provided the worldview, explained the origin of the world, the reason of many, if not all, natural, social, and psychological phenomena. Religion and religious practices were also a source of solace, because they provided a way of grappling with life's exigencies. However, the world of the gods was hardly one of exemplary morality. Gods were subject to the same passions as humans, and they seldom had any scruples about fulfilling their desires, regardless of how repulsive they were. The divine sphere had a hold over the terrestrial realm, but it was of no compelling moral quality. The divine sphere explained the world, but the sphere itself was filled with infrequently repellent qualities. Theologically and philosophically it seemed necessary to some thinkers to retain the divine world as an explanatory principle, but also to make it more, well, divine, more sublime. The first question which would have to be asked was this: what is the essence of the divine, or: what are the most important attributes of gods? The most important attribute presented itself in a phrase used by Homer and Hesiod that they are living forever (αἰὲν ἐόντες, *Il.* 1.290, *Th.* 33). Gods are born, but they are immortal. This traditional view of living forever was strengthened by Anaximander's contemporary, Pherecydes of Syros, whose "philosophical fable"[21] opens with the statement that "Zas and Chronos were forever, along with Chthonie" (7B1), that is, there are three divinities that always existed, and they became the source of other gods; they will live forever, but they also always lived; they are truly eternal beings, without beginning nor end. Philosophically, a next step was needed, namely one that determined the essence of divine eternity, or at least the essence of immortality. Living forever, living for an infinitely long time, i.e., temporal infinity, constitutes the essence of divinity.[22] It was then the work of a philosopher to construct the concept of the divine principle of the world to be infinity, the *Apeiron*, in which infinity, having primarily the temporal aspect, is also extended to spatial dimension.[23] This infinitistic aspect of the *arche* appeared first in Thales whose *arche*, which is water, is infinite: "of those supposing the existence of one single element, some called it *apeiron* in respect to size, as Thales did for water" (11A13; cf. Aristotle, *Phys.* 203a1-4, 203a16-18).[24] However, the concept of the *arche* becomes more philosophically refined in Anaximander for whom the infinity itself becomes the *arche*.

---

[20] Maybe a trace of such a possibility is the fact that some sources report the earth to have cylindrical shape (A10, A11) and some say the earth was spherical (A1): it can be both – in different universes.

[21] As it is called by Kahn, *op. cit.*, 171.

[22] This aspect of divine essence is expressed by the apophthegm attributed to Thales, "What is the divine? That which has no beginning nor end" (DL 1.36). The concept of the divine also includes the source of cosmic order, the moral order in particular, Adam Drozdek, *Greek philosophers as theologians*, Aldershot: Ashgate 2007, 1-2, 10.

[23] This temporal aspect of original infinity is recognized by the authors who propose that Chronos, an all-embracing Time-divinity was the model of Anaximander's *apeiron*, Theo G. Sinnige, *Matter and infinity in the Presocratic schools and Plato*, Assen: van Gorcum 1971, 5-6. The *apeiron* is even reduced to "the unending succession of generation and destruction" by Elizabeth Asmis, What is Anaximander's *apeiron*?, *Journal of the History of Philosophy* 19 (1981), 283, and it is stated that Anaximander never used "the *apeiron*" as a noun and he only spoke about "infinite Chronos, eternal and unaging," А.В. Лебедев, ΤΟ ΑΠΕΙΡΟΝ: не Анаксимандр, а Платон и Аристотель, *Вестник древней истории* 1978, no. 2 (144), 45. Time is also considered to be an entity coeval with the *apeiron*, the former being an area of activity if the latter, Hermann Schmitz, *Anaximander und die Anfänge der griechischen Philosophie*, Bonn: Bouvier 1988, 26-31, 72. Cf. Hermann S. Schibli, *Pherekydes of Syros*, Oxford: Clarendon Press 1990, 30-33, 37.

[24] The testimonies are not incontrovertible, R.M. Dancy, Thales, Anaximander, and infinity, *Apeiron* 22 (1989), 155-156, 183-186.

1.4 NAMING THE *ARCHE*

For the name of his *arche*, Anaximander chose the word which conveys the meaning of infinity, although there is some uncertainty about the primary meaning of the word or rather the meaning of the simplex that follows the α- privative. If it is πέρας (πεῖρας, πεῖραρ), limit or end, then ἄπειρος means endless, unlimited, limitless, boundless.[25] The meaning of infinity is not incompatible with boudlessness, but when the epic poets spoke about γαῖα ἀπείρων (*Il*. 7.446, 22.342, 5 times in the *Odyssey*, *Th*. 187, *Op*. 160, 487)[26] or ἀπείριτος (*Th*. 878) or ἀπειρεσίη (*Il*. 20.58), or about πόντος ἀπείριτος (*Od*. 10.195, *Th*. 109) or ἀπείρων (*Th*. 678, *Od*. 4.510, Theognis 237), they spoke about the boundless earth and boundless sea, but in the sense of "very large" rather than "infinite."[27] This is because both earth and sea are said to have limits (*Th*. 736-738, 807-809); obviously, Hellespont does too, although it is described as ἀπείρων (*Il*. 24.545) and so is the small island of Delos (ἄπειρεσίη, Theognis 8). The word is used in that sense to describe the size of ransom, ἄπερείσι' ἄποινα (11 times in the *Odyssey*) and of gifts (ἄπερείσι' ἔδνα, *Il*. 16.178, *Od*. 19.529). For this reason, it can be suggested that the α- privative negates the root *per* used in verbs indicating forward movement (πείρω (fut. περῶ), περαίω, περάω) and thus ἄπειρος means "what cannot be passed over or traversed from end to end,"[28] or simply intraversable.[29] If so, the poets took "intraversable" to mean very large, impossible to traverse by humans (but possible to traverse by the gods), or impossible to traverse in a reasonable amount of time. It remained for a philosopher to drop possible qualifications and take boundlessness or impossibility to traverse in a literal sense, which amounts to ascribing to the *apeiron* the meaning of infinity, true infinity, never ending, having no limits at all. *Apeiros* as boundless has primarily if not solely the spatial character;[30] i.e., infinite space is meant. *Apeiros* as intraversable encompasses both the spatial and temporal aspects of infinity: something is so vast that the time needed to traverse it would never end, i.e., something intraversable is infinitely large, since it requires infinite time to traverse it.

Other meanings associated with *apeiros* are related to the meaning of infinity. It may mean countless when spoken about goats (αἶγες ἀπειρέσιαι, *Od*. 9.118). An association of *apeiros* with circularity is also proposed, since the word is supposedly connected with περί, which implies circularity.[31] However, whether the word *apeiros* "actively in itself had this sense is doubtful … no incontrovertible example of it can be produced."[32] So, *apeiros* also describes a circular or spherical object; in this case its "meaning is still something like 'uninterrupted,' and the notion of circularity is contained in the nouns" described by the adjective *apeiros*[33] and if 'uninterrupted' is taken in an absolute sense – never interrupted – then we have temporal infinity, particularly in the case of circular movement.

The etymologies just discussed have an ontological character since they pertain to what being *apeiros* is. Tannery proposed an epistemological etymology which is related to how *apeiros* can be known. According to this view, *apeiros* is derived not from πεῖρας, but from πεῖρα and means "that which cannot be experienced."[34] Tannery admitted that had it been the case, this meaning would have been specific only to Anaximander and was lost afterwards.[35] This etymology is highly hypothetical, but if we assume it to be true, then we must ask, why the *apeiron* would be considered the *arche* which cannot be experienced. Is it experienced at all because its nature surpasses the nature of the physical world? Or is it not fully experienced because it is too vast to be so experienced by any human? It would be inexperienceable if it were infinite, and thus, the epistemological meaning would be secondary to the ontological understanding.

1.5 ANAXIMENES

Anaximenes, a younger contemporary of Anaximander, expressed many similar views. Like Anaximander, Anaximenes said that "the underlying nature is one and infinite, but not indeterminate as he [Anaximander thought], but determinate, saying that it is air" (13A5). According to Anaximenes, "the principle of all things is air and it is infinite in respect to size but determinate in respect to qualities round about it; all things come from it by condensation or rarefaction" (A6); Anaximenes' "infinite air is the principle from which everything comes, what is, what was, and what will be, gods and things divine (A7) and this principle itself is divine (A10). The testimonies mention infinite air (A9, A10) along with the fact that air is always in motion (A10), and "motion has always been in existence" (A6). Moreover, air envelops the whole universe (B2).

---

[25] That apeiron can be viewed as the negative of *peras* was clear for Pythagoreans and for Aristotle, *Phys*. 203b7-8, 207a1-15.

[26] ἄπειρος has the following epic variants: ἀπείρων, ἀπειρέσιος, ἀπερείσιος, and ἀπείριτος.

[27] So large "that their boundaries were never reached," Seligman, *op. cit*., 34.

[28] Kahn, *op. cit*., 232; Peter J. Bicknell, Τὸ ἄπειρον, ἄπειρος ἀήρ, and τὸ περιέχον, *Acta Classica* 9 (1966), 39; Conche, *op. cit*., 71.

[29] Friedrich Solmsen, Anaximanders infinite: traces and influence, *Archiv für Geschichte der Philosophie* 44 (1962), 122; Solmsen pointed to Aristotle's attempts to explain the *apeiron* as something that cannot be traversed (*Phys*. 3.4).

[30] H.B. Gottschalk, Anaximander's apeiron, *Phronesis* 10 (1965), 51.

[31] Michael Kaplan, ''Απειρος and circularity, *Greek, Roman and Byzantine Studies* 16 (1975), 127; the possibility of associating *apeiros* with circularity is unfavorably mentioned by Aristotle, *Phys*. 207a2-7.

[32] Kaplan, *op. cit*., 140.

[33] Gottschalk, *op. cit*., 53.

[34] Paul Tannery, Pour l'histoire du mot 'ἄπειρον', *Revue de philosophie* 5 (1904), 704. The same etymology is accepted by Tumarkin who translated *apeiron* as inexperienceable (*unerfahrbar*) and the *apeiron* as the unknowable (*das Unerkennbare*), Anna Tumarkin, Der Begriff des ἄπειρον in der griechischen Philosophie, *Annuaire de la Société Suisse de Philosophie* 3 (1943), 56-57. Actually, the etymology was proposed before Tannery, see Charles S. Halsey, *An etymology of Latin and Greek*, Boston: Ginn 1893 [reprint, New Rochelle: Caratzas 1983], 114: *apeiros* – "without trial or experience of, ignorant of."

[35] Tannery, *op. cit*., 705.

Anaximander and Anaximenes agreed that there is one *arche*; it is infinite, divine, and the source of all things that envelops everything that exists. For Anaximenes the *arche* was air, a determinate substance, and for Anaximander it was the indeterminate *Apeiron*. Also, Anaximenes' mechanism explaining the emergence of all that exists is different from than Anaximander's separation-off and return, namely, the process of rarefaction and condensation. Importantly, both the *Apeiron* and air are spatially and temporally infinite. As sources of all that exists, both *archai* allow for an infinity of generated things, and, explicitly in Anaximander, an infinity of worlds. As divinities, both *archai* are probably infinite in a power that is able to generate anything that can be conceived by them. Theologically, Anaximenes closely followed Anaximander; ontologically, he softened Anaximander's *arche* and made it closer to Thales' ideas and to daily experience.[36] As to physics, he proposed an original cosmic mechanism for generating and disassembling world entities. But like Anaximader's mechanism, Anaximenes' proposal allowed for an eternal process of generation and degradation of things, for a temporal infinity of life in a spatially infinite universe through a never-ending succession of finite beings.

---

[36] "Anaximenes was unable to imagine Anaximander's bold abstraction – the *apeiron* as kind of empty, all-embracing primal space (*Urraum*) – and returned to Thales by choosing as the *arche* one of great parts of the world," Joachim Klowski, Ist der *Aer* des Anaximenes als eine Substanz konzipiert?, *Hermes* 100 (1972), 142 (for Klowski, Anaximander's *apeiron* was not a substance); Adam Drozdek, Anaximenes: theology and physics, *Eranos* 102 (2004), 43-44.

2 THE PYTHAGOREANS

We have very little information about Pythagoras. Most sources about his life and views come from centuries later. Except for very short references to Pythagoras by early authors (in particular, by Heraclitus, Xenophanes, Empedocles and Plato, who refers to Pythagoras by name only once, *Rep.* 600b), Aristotle is the earliest source. However, his reports have to be treated with caution since he was primarily interested in presenting the view of Pythagoreans in light of his own philosophy. He spoke about the Pythagoreans rather than about Pythagoras, and it is quite likely that he meant the Pythagoreans of his time, but quite possibly also early Pythagoreans.

2.1 HARMONIOUS UNIVERSE

The Pythagoreans observed that concordant musical intervals can be expressed through numerical proportions. A basic musical interval is an octave. If a half AC of a string AB is vibrating, then its sound is higher by one octave than the sound of the string AB. Therefore, the ratio for the octave is 2:1= AB:AC. If CB is the vibrating part of AB and CB is two thirds of the entire string AB, then the sound of CB is a fifth of AB, that is, the sound of CB is higher by a fifth (or by three and a half tones) from the sound of AB. Thus, the ratio for the fifth is 3:2 = AB:AC. Continuing, the ratio for the fourth is 4:3, and for the third, 5:4. Such proportions were derived empirically, probably by using a monochord with a movable bridge (*kanon*), for which the Pythagoreans were scorned by Socrates. He said that some "students of harmony ... waste their time like astronomers, measuring audible concords and sounds against one another," and Pythagoreans are guilty of the same approach (Plato, *Rep.* 531ab). However, it seems that there was a great deal of theory involved in such considerations as well, because a measurement error for more complex proportions can very easily make it difficult to decide whether the proportions are expressed by integers or not, as, for example, proportions 28:27, 32:27, or 243:224 (these proportions were given by Archytas, ca. 380 BC, see 47A16).

The Pythagoreans discovered an invariable element in music that is encoded in numbers and in numerical ratios. The material substratum is less important; the strings can be of different material, thickness, length, durability, but they will sound harmoniously if they are used in accordance with numerical ratios. The material substratum is needed for the instantiation of eternal and immutable numerical laws. These laws are the essence of reality; they constitute the principle of being, and the material substratum is of secondary importance because even its existence is derived from a number.

The harmony of music is only one step away from the harmony of celestial spheres. In both cases, invariable numerical proportions were observed, so that acoustics and astronomy became "sister sciences" (Plato, *Rep.* 531a). The universe was similar to an instrument with orbits being circular chords; this resulted in the harmony of the spheres. There was a disagreement about which body produces which sound, except for the moon, the closest to the earth producing the lowest sound, and the sphere of stars, the furthest from the earth, producing the highest note.

The importance of number, proportion, and harmony as expressed by numbers seemingly cannot be squared well with infinity. This is expressed by the Pythagoreans in the so-called table of opposites in which infinity is associated with the bad and finitude with the good (58B5). Does infinity really have a negative status in the Pythagorean universe?

2.2 LIMITERS AND UNLIMITEDS

According to the Pythagoreans, the world as we know it has a beginning. A brief report provided by Aristotle states that according to them, "when the one had been formed – whether from planes or surfaces or sperm or from principles even they themselves cannot account for – instantly the nearest parts of the infinite (*apeiron*) were drawn in [as a breath] and defined by the definite (*peras*)" (*Met.* 1091a15-18 = 58B26). Also, Pythagoreans "think that number is the principle both as matter for things and as forming their modifications and states and that the elements of number are the even and the odd, the latter of which is the limited (*to peperasmenon*), the former is the infinite (*apeiron*); the one is from both of these [elements], for it is both even and odd, and from the one [proceeds] the number and, as already stated, the whole heaven is numbers" (B5). That is, the order of creation is as follows: first, the one is created out of the *apeiron* and the limited/definite, then numbers out of the one, and then the world out of numbers.[1] The one comes from the principles that are simply assumed to exist, and in that sense they are unaccounted for: the *apeiron* and the limited always existed, because they were two principles required by the Pythagoreans to create the world. In the process of creation, after the one is created, the part nearest to the one is drawn in and used up to create numbers. Afterwards, the world is created, through the use of the definite. Because only the "the nearest parts" of the *apeiron* were used, the *apeiron* remains the *apeiron*, that is, the remaining part is still infinite and presumably fills the infinite space outside the heavens. Thus, the Pythagoreans can hold that "what is outside the universe is infinite" (B28).

Why was the one created first? Did a cosmic accident lead to the creation of the first number and then, from it, all other numbers? Hardly. For Pythagoreans, for whom the question of harmony – harmony of music as well as of celestial spheres – was so critical, the accidental emergence of a harmonious universe is hardly thinkable. Therefore, an organizational power also has to be assumed. This power was implicitly assumed by Anaximander as an attribute of his *Apeiron*

when he spoke about the *Apeiron*'s governing of the universe. In the monistic view offered by Anaximander, there was only one principle at the beginning, namely the *Apeiron*. The Pythagoreans assumed the existence of at least two principles as material causes of the world, but the organizing power, thereby a final and efficient cause, is yet another primal *arche*, namely the *harmonia*. This is explicitly stated by Philolaus according to whom preexisting principles, the limiters (περαίνοντα) and unlimiteds (ἄπειρα), are "not alike or of the same kind; it would have been impossible for them to be ordered if harmony had not supervened" (44B6). There are thus three principles: limiters (or the limited/definite), unlimiteds (or the infinite), and the *harmonia*. The first two provide material for the universe, the last one is a seat of rationality that organized the first two principles into a meaningful, well-fitted cosmos.[2]

The two first Pythagorean principles Aristotle mentioned are the limited/definite (*peperasmenon/peras*) and the infinite. They correspond to the two principles of Philolaus, the limiters and the unlimiteds, both of them in plural. Therefore, the limited and the infinite are really compound principles composed of single limiters and single unlimiteds. The *harmonia*, on the other hand, is in singular. There are then, in fact, three classes of principles, two of which contain multiple singular principles, and the third which contains only one principle, the *harmonia*. How many limiters and unlimiteds are there? The number is nowhere specified, but regardless of their number, they allow for constructing infinitely many different entities, in particular, numbers.

The compound character of the *apeiron* can shed light on a difficult explanation given by Aristotle: "the Pythagoreans also said that there was a void and that it enters heaven from the unbounded breath,[3] [heaven] also inhaling this void which separates the natures [of things], the void being some kind of division and separation of things [that come] one after another. This holds first in the numbers, for the void separates their nature" (*Phys.* 213b22-27 = 58B30). Can it be that the void is a limiter that serves as a separator of numbers? Apparently, it is separating them, not building them. However, the void is one of unlimiteds that separates individual entities in the world. Pythagoras said, according to Aristotle, that "the heaven is one and from the infinite are brought in [into heaven] time, breath, and the void that always separates the spaces of all [things]" (fr. 201 = 58B30). That is, time, breath (air?), and the void are included in the *apeiron*; they are unlimiteds brought into the world, and the role of the void is specified as a separator. By analogy, is time to serve as a separator as well? If the void is a spatial separator, time would be a temporal separator. Thus, and first, only numbers are created and separated with the void, then, as creation progresses, the numbers constitute entities in the world which are also separated by the void and, we should probably add, by time. separation, however, is very much like delimiting; that is, the spatially infinite void and temporally infinite time, two of the unlimiteds, act like limiters.

---

[1] Cf. James A. Philip, *Pythagoras and early Pythagoreanism*, Toronto: University of Toronto Press 1966, 50-51, 61, 66-68.

[2] On rationality and divinity of the *harmonia*, see Adam Drozdek, *Greek philosophers as theologians*, Aldershot: Ashgate 2007, 66-70.

[3] Retaining πνεύματος instead of the emendation πνεῦμά (Diels) or πνεῦμα (Heidel).

Since time, breath, and the void are explicitly listed by the Pythagoreans as unlimiteds, and fire is implicitly mentioned by Philolaus as an unlimited (44B7),[4] it appears that an unlimited is an entity that is infinitely extended spatially or temporally. It appears obvious that time is infinite and so is the void (space), but also, an infinite store of air and fire can be assumed to exist. The *apeiron* would be a collective name for all these infinite entities with one common property, the infinity.[5] This infinity is their primary characteristic, so it is not contradictory to their nature that they may limit one another. Before the creation of the world and beyond the heavens after creation infinitely extended unlimiteds exist. Are they mixed together? Possibly. Instead, they may be divided into separate zones; hence there would be borderlines, or limits, between the two adjacent unlimiteds. Their primary character of being unlimiteds would not thereby be seriously compromised. The fact that they limit something – like the void is limiting objects under the heavens – is secondary to their nature and, in a way, a happy accident. True limiters, on the other hand, cannot function as unlimiteds.

How about a rare substance such as gold? Is it a limiter or an unlimited? If an infinite amount of gold exists beyond the heavens, it is an unlimited. But is there an infinite amount of gold? And what if it is not? Then it would be a limiter, however, limiters could be on different levels, i.e., one limiter can limit another limiter, and thus, categorizing a finite amount of original gold would not change much in the Pythagorean worldview.[6] It is also possible that gold beyond heaven is infinitely extended – and thus it is a true unlimited – and yet there is a finite amount of it. An adaptation of Zeno's argument can help here. Consider a cube of gold. The cube is cut horizontally into halves and the two halves are put next to one another. One of the two halves is cut horizontally in half and the upper piece is put next to the bottom piece. By continuing this process indefinitely, we would get infinite golden stairs of finite volume – an unlimited.

The Pythagoreans asserted that "the *apeiron* is the even; for this [even], divided off and limited by the odd, brings to things their unlimitedness (ἡ ἀπειρία); an illustration of this is what happens to numbers; for when the gnomons are

---

[4] Carl Huffman, *Philolaus of Croton*, Cambridge: Cambridge University Press 1993, 41-42.

[5] And only in that sense it would be true that "air and the void are, strictly speaking, only manifestations, each in its particular field, of the primary Unlimted," J[ohn] E. Raven, *Pythagoreans and Eleatics*, Chicago: Ares Press 1981, 49. Abraham referred to the table of opposites when he stated that "all entries under Limit are limiters, and all entries under Unlimited are unlimiteds, while the things which Philolaus says come from both limiters and unlimiteds would arise from combinations of items in both columns of the Table of Opposites," W. Emmanuel Abraham, Did the Pythagoreans use recursive definitions and functions?, in, K.I. Boudouris (ed.), *Pythagorean philosophy*, Athens: International Center for Greek Philosophy and Culture 1992, 17. However, with this characterization it is not clear what constitutes a distinguishing feature of unlimiteds.

[6] If an unlimited is considered to be a continuum (Huffman, *op. cit.*, 47), then the scope of unlimiteds considerably increases. In this way, each piece of gold in the non-atomistic universe of the Pythagoreans would be an unlimited as would be each interval of time, or each volume of air. The unlimiteds would thus be inextricably linked to limiters, and so the pure essence of their apeironic nature would be considerably undermined.

placed around the one and separately, in the one case the figure always changes, in the other case [the figure] is one [and the same]" (58B28).

A typical explanation[7] of this obscure statement is as follows: when starting with one dot (or a pebble) represented in the left diagram by 1:

3 3 3   3 3 3 3
2 2 3   2 2 2 3
1 2 3   1 1 2 3

and adding a gnomon (an instrument used for measuring right angles)[8] consisting of three dots represented by 2s, we have a square; after adding another gnomon of five dots represented by 3s, the resulting figure is still a square. It will remain a square when continuing to add gnomons in the indicated fashion. When starting with two dots, as in the right-hand side diagram, and adding a gnomon of four dots represented by 2s, we have a rectangle with proportion of sides 2:3; adding another gnomon of six dots represented by 3s gives us a rectangle with different proportion, 3:4. That is, when adding gnomons with an odd number of dots, we always obtain squares; when adding gnomons with an even number of dots, the shapes of rectangles differ by having differently proportioned sides, and there is an infinite number of possibilities. This may very well be the argument Pythagoreans used, but the argument can easily by thwarted by taking a different starting point; e.g., three dots and using gnomons with an odd number of dots, as in:

3 3 3 3 3
2 2 2 2 3
1 1 1 2 3

In this example, adding an odd number of dots to an existing rectangle results in rectangles with different proportion of sides: 1:3, 2:4, 3:5, etc., whereby odd numbers appear to infuse things with unlimitedness.

This description is interesting in that the operation of limiting can bring unlimitedness, if only as an unwanted side-effect. That is, the operation of limiting something may spawn into unlimitedness, limiting turns into its opposite; i.e., the borderline between limitedness and unlimitedness is quite fluid. Unlimitedness, or infinity, is ubiquitous; it can result, or maybe just be brought to light by the very operation that tries to limit it.

However, an argument for ubiquity of limitedness can also be made. The void separates everything that exists, whereby an unlimited plays the role of a limiter.

2.3 NUMBERS

According to Aristotle, the Pythagoreans "construct the [whole of] nature out of numbers" (*De caelo* 300a16 = 58B38; Cicero, *Acad.* 2.118) and "bring/reduce all things to numbers" (*Met.* 1036b12 = 58B25). However, Aristotle also gave other characterizations of the Pythagorean view of numbers which may appear to be incompatible with the view that things are made out of numbers and so things are numbers (*Met.* 986a16, 987b28, 1083b11-12). He also stated that things are similar to numbers and are modeled after them (985b27,33), and elements of numbers are elements of things (986a1-2,17-18).[9] Even in the same paragraph he said that things exist as imitations of numbers (987b11-12), that numbers are causes of the substance of things (b24-25), and that things themselves are numbers (b28) and thus numbers are not separate from things (b29-31). In these characterizations, an emphasis is placed on the ontological aspect of numbers – numbers are part of reality – but an epistemological aspect is also present: numbers are models of things. The latter aspect was very clearly stated by Philolaus when he said that "all things that are known have number; for it is not possible for anything to be understood or known without it" (44B4). However, Philolaus also recognized an ontological aspect of numbers which is clear in the statement that there are two kinds of numbers, odd and even, and a third kind, the odd-even, which is "from both mixed together" (44B5). Numbers are mixtures of even and odd; that is, they are part of reality since a mixture cannot be understood purely epistemologically. Because "nothing could be known if everything were unlimited" (44B3), knowledge of numbers is insufficient to know the world. Limitation has to be part of reality, and saturating reality with numbers guaranteed the limitedness of the world and thus the possibility of knowing it. Numbers are epistemologico-ontological entities.[10] Things in the world can be considered as structures made out of proper proportions of the limiters and unlimiteds. The proportions themselves are numbers, proportions of limiters and unlimiteds, mixtures of limiters and unlimiteds in proper proportions that serve as materials to be molded by the *harmonia* to create the world. If preexisting limiters and unlimiteds are envisioned as sets, possibly infinite,

---

[7] Walter Burkert, *Lore and science in ancient Pythagoreanism*, Cambridge: Harvard University Press 1972, 33 note 27 with many references.

[8] For history of the term, see Thomas Heath, *A history of Greek mathematics*, New York: Dover 1981 [1921], 78-79.

limiters = { limiter$_1$, limiter$_2$, …, limiter$_i$, …}
unlimiteds = { unlimited$_1$, unlimited$_2$, …, unlimited$_j$, …}

then because the one is explicitly stated to originate from both the even and the odd (and thus from *peras* and *apeiron*, or limited and unlimited, for the one is both even and odd, *Met.* 986a19), then the one can be considered a set of homogeneous materials, each material consisting of only one limiter or unlimited[11]:

the one = {material(limiter$_1$), material(limiter$_2$), …,
material(unlimited$_1$), material(unlimited$_2$), …}

---

[9] Wilhelm Bauer, *Der ältere Pythagoreismus*, Bern: Steiger 1897 [reprint, Hildesheim: Olms 1976], 10; F.M. Cornford, Mysticism and science in the Pythagorean tradition [1922], in A.P.D. Mourelatos (ed.), *The Pre-Socratics*, Garden City: Anchor Books 1974, 136; W.A. Heidel, The Pythagoreans and Greek mathematics [1940], in D.J. Furley, R.E. Allen (eds.), *Studies in Presocratic philosophy*, New York: Humanities Press, v. 1, 362; Leonid J. Zhmud, *Wissenschaft, Philosophie und Religion im frühen Pythagoreismus*, Berlin: Akademie Verlag 1997, 271-272; Huffman, *op. cit.*, 60.

[10] Pythagorean number "has two aspects, the one is that of abstract number, which belongs to their mathematics, the other is concrete number, which belongs to their ontology," Teruo Suzuki, Philolaus: epistemologist or ontologist?, in Boudouris, *op. cit.*, 212.

[11] The Pythagoreans started counting from two, so technically, the one is not a number.

Number 2 would be a set of materials which are mixtures of two different stuffs, the latter being limiters and unlimiteds:

number 2 = { material(1 part of limiter$_1$, 1 part of limiter$_2$),
material(1 part of limiter$_1$, 1 part of limiter$_3$),
…
material(1 part of limiter$_1$, 1 part of unlimited$_1$),
material(1 part of limiter$_1$, 1 part of unlimited$_2$),
…
material(1 part of limiter$_2$, 1 part of limiter$_3$),
material(1 part of limiter$_2$, 1 part of limiter$_4$),
…
material(1 part of limiter$_2$, 1 part of unlimited$_1$),
material(1 part of limiter$_2$, 1 part of unlimited$_2$),
…
material(1 part of limiter$_i$, 1 part of limiter$_{i+1}$),
material(1 part of limiter$_i$, 1 part of limiter$_{i+2}$),
…
material(1 part of limiter$_i$, 1 part of unlimited$_1$),
material(1 part of limiter$_i$, 1 part of unlimited$_2$),
…
material(1 part of unlimited$_j$, 1 part of unlimited$_{j+1}$),
material(1 part of unlimited$_j$, 1 part of unlimited$_{j+2}$),
… }

It may be assumed that not all combinations of different limiters and unlimiteds constitute materials in this set. For instance, it is very likely that there is no material made out of water and fire.

Generally, any positive integer k is, ontologically, a set of materials, each material containing in total k different limiters and unlimiteds:

number k = {material(1 part of limiter$_1$, k-1 parts of limiter$_2$),
material(1 part of limiter$_1$, k-1 parts of limiter$_3$),
…
material(1 part of limiter$_1$, k-1 parts of limiter$_j$),
…
material(1 part of limiter$_1$, 1 part of limiter$_2$, k-2 parts of limiter$_3$),
…
material(1 part of limiter$_1$, k-2 parts of limiter$_2$, 1 part of unlimited$_1$),
…
material(k_1 parts of stuff$_{i1}$, k_2 parts of stuff$_{i2}$, …, k_s parts of stuff$_{ip}$),
… }

where $k = k_1 + k_2 + ... + k_s$ and $i_1, i_2, \dots, i_p$ is any sequence of numbers used as indices for different limiters and unlimiteds indicated above as stuffs.[12] The proportions indicated in each material are meaningful if the amount of particular material is finite. This is sufficient since Pythagoreans seem to have recognized the existence of one world only (unlike, say, Anaximander). However, to also include a possibility of infinite amounts of materials, proportionality in a particular material can be understood as proportionality in any finite sample of this material. Also, since time is one of the unlimiteds, time is an ingredient of each material. In this case, for example, material(1 part of time, 2 parts of earth) could be considered less durable than material(2 parts of time, 2 parts of earth). To allow for the possibility of eternal existence of some entities – souls in particular – some materials would not include time.

---

[12] A similar understanding of Pythagorean numbers seems to be given by the statement that "every substance that can be perceived by our senses appears in a certain numerical arrangement," S. Sambursky, *The physical world of the Greeks*, New York: Macmillan 1956, 28.

These numbers are created from the one, and then used to create anything else in the world so that any particular object would be an offshoot of some number *k*, or a structure properly molded by the *harmonia* from the number-material *k*. That is, when numbers are envisioned as sets of uniformly proportioned materials, then the world and things in it are constructed of out numbers; that is, elements of numbers – particular materials – are elements of things; that is, everything can be reduced to numbers; that is, things are numbers. Because numbers are also epistemological entities and part of a cognitive apparatus residing in the minds of particular people and of the *harmonia* that shaped the world, things are *similar* to numbers, imitations of numbers, modeled after them, whereby cognition is possible.

Infinity has a pronounced position in the Pythagorean world; it is an indispensable principle that is the material from which the world is constructed. Because of its unboundedness, infinity has to be shaped by the *harmonia* with limiters. Because of the need of mastering the *apeiron*, infinity has a somewhat negative status in Pythagoreanism as a necessary evil without which the world would not exist. But the very fact of indispensability of infinity in formation of the world infuses it with positive value. There is more. Numbers are the pinnacle of orderliness in the Pythagorean universe. How many numbers are there? An infinity. Is it a positive sign that there are infinitely many good things, or is it too much of a good thing? In this context, it seems, infinity is very positive, the more numbers exist, the more refined the harmony of the world can become. This requires, however, an infinite knowledge when shaping the world to find the best combination of numbers to be used. That is, the infinity of knowledge of the shaper of the world, the *harmonia*, is certainly positive. Infinity is not just an evil ontological monster to be tamed, but also a welcome property that enables the taming itself. The positive aspect of the *apeiron*, so pronounced in Anaximander,[13] is retained by the Pythagoreans.

---

[13] "Pythagoras was a younger contemporary of Anaximander, by whose thought he was probably influenced during his formative years and his maturity in Samos," Philip, *op. cit.*, 69. "The unmistakable influence of Miletus" on Pythagoras is recognized by Charles Kahn, *Anaximander and the origins of Greek cosmology*, New York: Columbia University Press 1960, 206.

3 XENOPHANES

Xenophanes is reported to have been a pupil of Anaximander (21A1) which is very likely considering that there is a short distance between Colophon and Miletus and the reputation of Miletus as an intellectual center. The influence of Anaximander is reflected in Xenophanes' interest in the search for the *arche*, his interest in rectifying the image of the divine and in the role of infinity in his investigations.

3.1 INFINITE WORLD

According to Xenophanes, "everything is from earth and everything comes eventually [back] to earth" (B27, A32, however, A36). Aristotle specifically said that no philosopher considered earth as the *arche* (*Met.* 989a5). It is possible, however, that Xenophanes' views were not mentioned since he viewed Xenophanes as a theologian rather than a natural philosopher or as someone for whom physical analyses were insufficiently important.[1] Aristotle also mentioned that no one considered earth as the *arche* for the soul (*De anima* 405b8) which apparently would belie the validity of the testimony that according to Xenophanes, the soul is of earth and water (A50). However, the testimony mentions two *archai*, earth and water, and actually, the two *archai* are also mentioned in a more general context when stating that for Xenophanes, "earth and water are everything which comes into being and grows" (B29, B33). In this way, Aristotle's denial that anyone considered earth as the *arche* means, literally, that no one considered earth as a single *arche*, i.e., he meant only monists, but this does not contradict the fact that Xenophanes believed in a pair of *archai*, earth being one of them.[2] Besides, Aristotle did mention the existence of the view that allows earth to be the *arche* (*Phys.* 193a21).[3]

An interesting view expressed by Xenophanes' own words is that the earth is extended into infinity downward (B28, A32, A33, however, A47).[4] According to the Milesians, the earth was a disk floating on water (Thales), suspended in the middle of the world (Anaximander), or supported by air (Anaximenes). One question which could arise was, what supported the support: if the earth floats on water, how deep is the water? – if on air, how deep is the air under the earth? Infinite? If so, then the earth can support itself by being infinitely extended downward, and that is the solution Xenophanes proposed. For the Milesians, the earth was a disk; was it then an infinitely extended disk resulting in an infinite column? This would be a precarious solution. If air were around the column so could be winds and they could tilt the column so that everything from the surface of the earth would slide into the abyss. But never was such tilting experienced, so maybe there was no air below the horizon (A33) and so no winds. But this still would not guarantee the solidity of the column. It would have to be from solid rock for rigidity but even then, an infinite column would have an infinite weight and it would crumble under its own weight. Therefore, it seems that Xenophanes' earth is also infinitely extended sideways.[5] Xenophanes is said to have spoken about multiple worlds that sink into the sea to become mud and then be regenerated again (A33). These worlds can be considered as appearing in consecutive order, one after another.[6] But it also can be a reference to multiple concurrent worlds perishing and regenerating independently. In fact, one testimony ascribes to Xenophanes a belief in infinity of worlds "in every direction," i.e., concurrently (12A17). Another testimony states that according to Xenophanes there is an infinite number of nonoverlapping worlds (21A1).[7] If the earth is assumed to extend sideways, there is enough room for multiple worlds on its surface, in fact, for infinity of worlds. In this way, Anaximander's infinity of worlds in the entire space would be folded into infinity of worlds on the infinite surface of Xenophanes' earth, the worlds being separated by seas, i.e., by water, the second Xenophanian *arche*. In favor of the possibility of infinite concurrent worlds testifies the fact that, according to Xenophanes, there are infinitely many suns and moons (A33). It is possible that an infinity of suns and moons perishing and born every day is meant here – one testimony states that during its setting, the sun is extinguished, and another sun rises in the east (A41) – but it is also possible to understand this as concurrent infinity: infinitely many suns and moons, one per each world; in fact, Xenophanes is said to have maintained that there are many suns and moons throughout the regions, sections and zones of the earth (A42a); however, both interpretations are possible: in all these infinitely many worlds, the suns and the moons are extinguished and regenerated every day and night through eternity.

---

[1] Karl Deichgräber, Xenophanes περὶ φύσεως, *Rheinisches Museum* 87 (1938), 13; Adolf Lumpe, *Die Philosophie des Xenophanes von Kolophon*, München: Foth 1952, 41; J.H. Lesher, Commentaries, in Xenophanes, *Fragments*, Toronto: University of Toronto Press 1992, 127.

[2] Lumpe, *op. cit.*, 41; Lesher, *op. cit.*, 127. Cf. Jaap Mansfeld, Theophrastus and the Xenophanes doxography, *Mnemosyne* 40 (1987), 294.

[3] The earth-and-water interpretation is advocated by Lumpe, *op. cit.*, 42-43 and Lesher, *op. cit.*, 133. The mention in B27 of earth alone can be explained by the fact that Xenophanes meant the earth which is under our feet, the soil, rather than earth as the primal element, Lesher, *op. cit.*, 127.

[4] Aristotle impatiently commented that Xenophanes made this claim to avoid searching for causes, *De caelo* 294a23-24 = A47, which may indeed be true, but Aristotle himself is just as

3.2 INFINITE GOD

The dual *archai* modify the approach of the Milesians who proposed one single *arche*. In fact, Xenophanes went even further by including the third *arche*, which was God. There are thus three *archai* in Xenophanes' universe: two of physical nature and one of theological character.

Xenophanes criticized Homer and Hesiod for their anthropomorphic image of the gods and the illicit behavior ascribed to them in the epic poems (B11, B12), for a popular supposition that the gods are born (B14, A12) and for their appearance in which they resembled humans (B15, B16). In his opinion, there is only one God "not at all like mortals in body or in thought" (B23) who is a living being that perceives and thinks through his entire being (B24, A32), effortlessly controls the entire universe by his mind (B25), who is immobile (B26), unborn and eternal (A34).

First, where is this God? He is a body of different nature than the body of mortals and thus occupies space in which he is immobile. Apparently, God is extended above the earth, somewhere in the air, although it is not impossible to see God as penetrating earth as well if his body is so fine that such penetration is possible. It is much later, in the universe of the Stoics, when an interpenetration of bodies becomes uncontroversial. In this case, Clement can be excused for having thought that Xenophanes' God is incorporeal (*Strom.* 5.109 = B23).

How big is God? Xenophanes makes no statement concerning magnitude of God or, as stated by Aristotle, he was not altogether clear on the issue nor sufficiently scrutinized it (*Met.*986b22-23 = A30).[8] This may be the reason why testimonies are inconsistent. Xenophanes' God was said to be infinite (A31, A34), but also limited (A35) and yet the author of *De Melisso Xenophane Gorgia* (*MXG*) pronounced Xenophanes' God to be "neither infinite nor limited" (977b3). However, this is not necessarily Xenophanes' opinion but rather the opinion of the author of the *MXG* about attributes of Xenophanes' God which is clear from the sentence that follows and states that "what is not is infinite, for it has neither middle nor beginning and end, nor any parts, and such is [the nature of] the infinite; but what is could not be like what is not," and the argument continues to show that finitude cannot be posited about God either (977b3-8). This is not Xenophanes' argument, this is how the peripatetic author of the *MXG* thought Xenophanes'

---

guilty with his assumption that the world exists from eternity thereby solving the problem of creation of the world.

[5] "The earth [which is] flat, has no limits, neither on the side, nor underneath" stated simply P[aul] Tannery, Histoire du concept de l'infini au VIe siècle avant J.-C., *Revue philosophique de la France et de l'étranger* 14 (1882), 630. Xenophanes "presents the earth having an infinite flat surface and infinite depth," the earth with "the double *apeiron*," Alexandre P.D. Mourelatos, La terre et les étoiles dans la cosmologie de Xenophane, in A. Laks, C. Louguet (eds.), *Qu'est-ce que la philosophie présocratique?*, Villeneuve d'Ascq: Presses Universitaires du Septentrion 2002, 347, 348.

[6] This interpretation is favored by Christian Schäfer, *Xenophanes von Kolophon*, Stuttgart: Teubner 1996, 132.

[7] φησὶ … κόσμους δὲ ἀπείρους, οὐ παραλλακτοὺς δέ. The meaning of παραλλακτός as changeable does not fit the context, but παράλλαξις can also mean overlapping as used by Hippocrates in the context of overlapping of broken bones (Lesher, *op. cit.*, 197 note 1), that is, in a spatial context, which points to infinity of worlds which are spatially nonoverlapping.

[8] More decisively: "Xenophanes did not provide God with spatial characteristics," Aryeh Finkelberg, Studies in Xenophanes, *Harvard Studies in Classical Philology* 93 (1990), 124.

God should be.[9] Similarly, in following sentences the author provided his own argument that Xenophanes' God is neither moved nor unmoved notwithstanding Xenophanes' own words that God is unmoved (977b8-17 = A28).

The claim that Xenophanes' God is limited (A35) is made even more specific by several testimonies stating that he is spherical (A1.19, A28.7,11; A31, A33.2, A34-36); however, these testimonies appear to confuse the views of Xenophanes and Parmenides which was easy to do since Parmenides was considered a pupil of Xenophanes (A30, A31).[10] It was also suggested that the attribute ὅμοιος ascribed by Xenophanes himself to God could have been the source of God's spherical shape. According to this explanation, Xenophanes "abandons the idea of coming-into-being and passing away but says that the whole is always alike (ὅμοιος)" (A32, cf. A33.2, A35), i.e., being alike meant immutability, but later, being alike was understood as being spherical when, for instance, stating that Xenophanes' God was "limited and spherical since he says that [God] is alike in every way" (A31).[11]

However, words of Xenophanes himself should take precedence over any testimonies. True, he never said how great God is, magnitude-wise, but, as already mentioned, he said that God effortlessly shakes all things by his mind (B25). The nature of shaking is not quite clear. It may very well be an allusion to Homeric Zeus shaking Olympus (*Il.* 1.528-530).[12] Maybe this shaking includes the ability of creating the universe or rather molding it from earth and water. This possibility is indicated by Xenophanes' statement that "if God did not make (ἔφυσε) yellow honey, [people] would consider figs much sweeter" (B38), i.e., the fact is announced that honey was made by God. Presumably, if God made honey, he also could have made other things. But it is certain that the shaking expresses God's ability to affect everything that exists, constructively or destructively.[13] But the earth itself is infinite. So, God has to have enough power to have an impact on an infinite universe. It seems that an image of a spherical, i.e., spatially limited God

is quite incompatible with an infinite universe, or rather with God's ability to influence this universe in all its parts at all times. Would a limited being be capable of reaching with his mind an infinitely extended world and do it eternally? Regardless of how large this divine sphere would be, it would be an insignificantly small speck in comparison with infinite expanses of the universe. This hardly gives confidence in the possibility that God could influence everything effortlessly. Therefore, it appears that Xenophanes' God has to be infinite, infinitely extended throughout the universe. Also, God's knowledge appears to be infinite, too. Traditional gods are very harshly criticized by Xenophanes from, primarily, an ethical point of view. That is, the One God is free of any unethical doing. Therefore, God's shaking has to be done in such a fashion that any wrongdoing is impossible. But because the universe is infinite, infinite knowledge is needed to decide which shaking is – if not immediately, then in the long run – beneficial to a particular world.

It is possible that this co-infinity of the world and God led to merging God with the world into unity, into the one, as expressed by some testimonies (A4, A35-37). What would constitute unity here would not be a pantheistic blending of God with the world, but the fact that both the world and God are infinite and thereby appear to be one. Infinity is the primary characteristic of the divine as expressed in Anaximander's *Apeiron*, a divine being. Because the world is infinite, it acquires an important divine attribute. It does not become God (e.g., the world is not intelligent, although there are intelligent beings in it), but through this attribute it has a divine aura. One step can be made by abolishing any differences between the world and God and proclaiming them to be one. This pantheistic step is, however, alien to Xenophanes. He did not criticize crude anthropomorphism to fall into a trap of identifying God with the world.

The pantheistic tendency of some doxographers may be caused by the fact that there is a curious cosmic status of God in Xenophanes' universe: there does not seem to be much connection between God and the world. For Anaximander, the *apeiron* is involved in creating and shaping the universe through mechanism of separation and destruction and then in the providential process of governing the world. For Anaximenes, the divine air is the starting point of molding the universe through the rarefaction and condensation mechanism. However, except for a side remark about honey, we learn nothing about the role of Xenophanes' God in creating the universe. Xenophanes offered various explanations of natural phenomena, but all of them were given in physical terms. God is apparently not involved. For example, we learn that the sun comes into being every day from small pieces of fire that are collected (A33.3), the moon is compressed cloud (A43), lightning is the result of the movement of clouds (A45), etc. He even dispelled traditional explanations of the action of the gods in nature by pointing to the natural character of physical phenomena; he said, for instance, "the one they call Iris, is by its nature a cloud that is seen as purple, reddish, and yellow" (B32); there is no supernatural agency here, it is just nature at work. Really, the only action that God exercises – or can exercise – in respect to the universe is shaking it. We learn nothing about the character of this shaking, but it could be potentially damaging.

---

[9] It is said that in this argument the author projected "the more complicated method of Zeno onto the argument of Xenophanes a priori falsified" and "applied incorrectly even Zeno's method," R. Falus, La 'démonstration' de l'idée de dieu chez Xénophane, *Acta Antiqua Academiae Scientiarum Hungaricae* 19 (1971), 246.

[10] Consequently, "from an early date Parmenidean notions were fathered upon the presumed founder of the Eleatic school," as expressed by Mansfeld, *op. cit.*, 304-305. See also Michael C. Stokes, *One and many in Presocratic philosophy*, Washington: Center for Hellenistic Studies 1971, 82-84.

[11] Július Špaňár, Bol Xenofanov boh σφαιροειδής?, *Listy filologické* 89 (1966), 243.

[12] Lumpe, *op. cit.*, 19; Lesher, *op. cit.*, 107, 109-110.

[13] The reference to shaking "was not dictated by the desire to define the modalities of divine action, but is explained solely by making a reference to a traditional representation [of the divine] that Xenophanes takes over and at the same time rectifies it." Speaking about shaking was a means of indicating "by a striking image the extent of the divine power which one intends to define." Xenophanes simply tried "to make us understand the incommensurability of his [God's] power," Daniel Babut, Sur la 'théologie' de Xénophane, *Revue philosophique* 164 (1974), 427. The shaking implies not only God's sovereignty but also "the invincibility and certainty of his judgment and plans," suggested Emese Mogyoródi, Xenophanes as a philosopher: theology and theodicy, in A. Laks, C. Louguet, *op. cit.*, 283.

Maybe because of that possibility people should not ignore God and are encouraged to worship him by singing hymns and by reverent words and pure speech (B1.13-14).[14] However, one way doxographers saw to make God more relevant to the world was to blend him with it, to pronounce him to be one with the world. Pantheism is a solution, but it certainly is not Xenophanes': "A thinker, who so decisively emphasizes that various celestial bodies and celestial phenomena are only cloud formations, cannot at the same time have identified the universe with God."[15]

Infinity is a divine attribute, but by itself it is insufficient for anything to be divine. The earth is infinite, both spatially and temporally, but it is certainly not divine for Xenophanes. God is also an intelligent being – earth certainly is not; God is also immutable – and there is nothing immutable on the surface of the earth in various worlds. They come and go in an eternal cycle of generation and perishing – an infinity of mutability, and for Xenophanes there is nothing divine in it.

Xenophanes' universe is saturated with infinity. There are two infinite physical *archai*, water and earth, to make possible for the existence of an infinitely extended earth with an infinite number of *kosmoi* on its surface separated by bodies of water. In these worlds, the suns and moons are generated every day from eternity to eternity. God, the third *arche*, is also spatially and temporally infinite with infinite knowledge.

---

[14] Schäfer, *op. cit.*, 180.

[15] Lumpe, *op. cit.*, 24.

4 HERACLITUS

Heraclitus seems to be the philosopher of change whose views are apparently encapsulated in the concise statement, *panta rhei*. He chose fire as the substrate from which everything originates, a fickle and unstable substance, which appears to confirm his view that "everything flows." However, change and instability perceived in the universe are just the surface beneath which stability and orderliness can be found.

4.1 ETERNITY

Heraclitus stated that "the cosmos, the same for all, no one of gods or men has made, but it always was and is and shall be, an ever-living fire (πῦρ ἀείζωον), kindling in measures and extinguishing in measures" (B30). The cosmos is an orderly arrangement of the world, the world order, the set of laws that regulate the course of natural phenomena including social and psychological phenomena. The world that exists is eternal, uncreated and imperishable. The same laws determine events in the world from eternity to eternity.

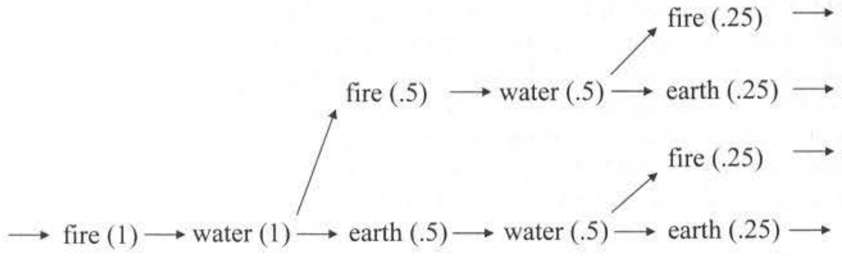
The concept of an uncreated and imperishable world was quite new in Greek philosophy and will reappear later in Aristotle. For the Milesians, for Pythagoreans, and, apparently, for Xenophanes, and, about the same time, for Empedocles and Anaxagoras, the world was clearly created from the eternal substrate by intelligent agency which was this substrate itself (the *Apeiron* or air) or by coeval agent (Mind). Although it was not always explicitly stated, the created world was a subject of destruction to be followed by another act of creation of the world which could be different from the world before it. This apparent changeability of the world was considered by Parmenides to be just an appearance, the way of opinion which stops at the surface of reality. Reality itself, Being, is eternal, immutable, and imperishable; motion is but an illusion; change is only a chimera. Such a view was pulled from the level of lofty and counterintuitive metaphysics to a more earthly level by Heraclitus who, in a way, merged the world of truth and the world of appearance.[1] There is motion, there is change, there is mutability in the world, but the change is regulated by laws that may not be obvious – and are not obvious – to every eye. Immutability of reality lies in the unchangeable character of the laws of change. There is no accident in the world; all changes are determined by unchangeable laws. The laws are eternal; they were not created by any earthly or superior being and they will never be abolished. The seat of these laws is fire that is not only eternal, but also ever-living since the essence of life and existence – the eternal laws – are in the bosom of fire.

---

[1] This statement is not intended to suggest that Heraclitus knew Parmenides' views.

The transformations of fire into other elements of the universe take place in measures, in orderly and proportional fashion. There are "fire's turnings (τροπαί): first, sea, and of sea half [becomes] earth and half burner. Sea is poured forth and is measured in the same proportion (λόγος) as it was before it became earth" (B31). It is suggested that *trope* is a pun on *tropos*, which means turning and also manner; therefore, it is possible that turnings of fire from B31 are also its "modes of existence."[2]

There are at least two other world masses – water and sea – two modes of fire's existence, and they can convert one to another. However, fire has a priority as an active, ever-living substance, initiating any transformation. Therefore, water is but a form of existence of fire. It is fire in a liquid form, apparently contradictory to its nature; after all, water extinguishes fire. However, at the deeper level, there is no principal difference between water and fire. This is far from strange from today's perspective since we know that a molecule of water consists of two atoms of hydrogen – which can burn – and an atom of oxygen – which maintains burning – and thus water, in fact, is a liquefied fire. The transformations are apparently performed proportionally so that only a half of water (sea) turns into earth and the other half back into fire. Also, earth turns into water (sea pours forth from, which is understood, earth) where proportionality of masses is also maintained (B31). These turnings can be understood in the twofold fashion, diachronically and synchronically. Through diachronic turnings a half of sea eventually dries up by being turned into earth, a half evaporates being turned into fire. It is not specified how much fire turns into water and, rather obliquely, how much earth into water. Let us assume: all fire turns into water, which seems to be specified for earth at the end of B31. Assuming that the turnings take place at the same rate, the following diagram can be formed (numbers in parentheses indicate volumes of substances, beginning with one unit of fire):



It is clear that at no point in time the world can be all fire. It could be all water, but not all fire, which hardly speaks in favor of the primal status of fire. This could be rectified by assuming that only a fraction of fire turns into water. Also, assuming different rates of turnings for different substances does not help either in an attempt to show that everything can turn back to fire, therefore, B31 can be taken as a statement that speaks against the possibility of ecpyrosis when the world periodically turns into fire only.[3]

---

[2] Joel Wilcox, *The origins of epistemology in early Greek thought: a study of psyche and logos in Heraclitus*, Lewiston: The Edwin Mellen Press 1994, 42.

The assumption of synchronic turnings renders the following diagram:

$$\text{fire} \underset{x}{\overset{.5}{\leftrightarrows}} \text{water} \underset{.5}{\overset{1}{\leftrightarrows}} \text{earth}$$

Synchronic turnings consist in maintaining the same mass of water all the time by the simultaneous drying up and evaporation of water on the one hand and, on the other hand, concurrent liquefaction of earth and fire.[4] Since half of water turns into fire and half of it into earth (indicated by .5 next to corresponding arrows), and the entire earth turns into water (indicated by 1), there has to be twice as much water as there is earth to make it possible. How much fire is there? There is no fragment that is specific on the issue. If entire fire turns into water (and thus, $x = 1$), then there would have to be twice as much water as fire, i.e., there would be 50% of water, 25% of fire and 25% of earth in the world, which, volume-wise, gives preeminence to water. However, it still could be claimed that fire is the primal substance since life and rationality lies in it and other substances are only the forms of the existence of fire. But to give preeminence to fire also in respect to the volume, x has to be less than .5, and the smaller it is, the larger the amount of fire is in the universe. Be that as it may, the important thing is that all of these turnings are not considered to be random. They are taking place according to preestablished laws, according to rationality built into the ever-living fire. The Heraclitean world has always been orderly and will be so for eternity to come.

The world is eternal, the same world order always existed and always will exist, there is one world without beginning or end. The orderliness of the world is expressed by the proportionality of exchanges between the substances of the world, and the proportionality can only be meaningful if the world is finite. If a substance had been of infinite amount, what would have been a half of it? Proportionality, therefore, implies spatial limitedness of the world, and doxographers say as much explicitly (DL 9.8; Simplicius, *In Phys.* 24.1). This proportionality, logos (B31),[5] becomes a prime element of rationality of fire and thus of the cosmos. Logos is its rationality, orderliness, regularity; the structure of the world, its organization is according to the Logos.[6] Logos is eternal and yet, in Heraclitus' universe, it is meaningful because of the spatial finitude of the universe. If the world is of temporal infinity, then it appears that temporal laws have to be expressed in finite terms, i.e., they have to determine the periodicity of phenomena. This is periodicity in the eternally existing universe and does not have to entail cosmic conflagration.

---

[3] See Charles H. Kahn, *The art and thought of Heraclitus*, Cambridge: Cambridge University Press 2001 [1979], 143. Kahn is one of few researchers who claimed for Heraclitus the view of cosmic conflagration (see also T[homas] M. Robinson, Commentary, in Heraclitus, *Fragments*, Toronto: University of Toronto Press 1987, 99). Usually, testimonies suggesting such a view are dismissed as distortion initiated by Aristotle and Theophrastus, G[eoffrey] S. Kirk, *Heraclitus: the cosmic fragments*, Cambridge: University Press 1970, 322-324, 327-329, 336.

[4] Kirk, *op. cit.*, 330.

[5] "Proportion should be considered as the fundamental aspect of logos," Ewald Kurtz, *Interpretationen zu den Logos-Fragmenten Heraklits*, Hildesheim: Georg Olms 1971, 62.

[6] "The natural world and the order in it" are "expressed as the Logos," this world being the ever-living fire, Kirk, *op. cit.*, 317.

The world is regular and orderly but we do not always see it. The order is hidden behind apparently random arrangement of phenomena, and more often than not, we perceive this randomness as essential to the cosmic order. This is expressed by an enigmatic statement that "life(time) (αἰών) is a kidding kid, playing *pessoi*; the kid's is the kingdom" (B52). The term *aion* signified eternity beginning at least with Plato (*Tim.* 37d; Aristotle, *De caelo* 279a22-28).[7] In Heraclitus' time, it meant lifetime, life force, and time.[8] It is time understood as "the force on which man depends having not even slightest control over it," as "Time which imposes limits being implicitly understood as having no limits."[9] This gives a very strong indication that *aion* may have been understood by Heraclitus as temporal limitlessness, as temporal eternity, thereby becoming associated with ever- (*aei*, the kindred word of *aion*)[10] living fire, as ever-living seat of orderliness.[11] In this way, B52 would express a conviction that the world is not just a collection of accidental events, but there are regularities according to which the events occur. These regularities may not be obvious since "nature loves to hide itself" (B123). We may have some idea about the existence of these regularities, the laws akin to the rules of the game of *pessoi*.[12] However, they may not manifest themselves through the apparent irregularities of visible phenomena. If they are there, they look more like laws applied by an inexperienced hand, by a child who does not posses perfect understanding even of the rules of a game.[13] The rules are thwarted by unsteady application, hence the result. This appears to be an eternal state of the universe: irregularity, chaos, and an apparent lack of order as though a child would rule over cosmic events, bringing everything to confusion by his inexperience. However, it takes an especially keen eye to see that the world events are not a child's play and such perspicuity is inaccessible to most people: "many people do not understand things they encounter, nor, when instructed, do they know them, although it seems to them [that they do]" (B17).

Eternity is the order, the order, i.e., Logos, is eternal. Only a few can see that, although, theoretically, everyone could. The cosmos with its structured arrangement is eternal, and this arrangement, the proportionality of exchanges of substances, guarantees that it will remain eternal. Eternity is thus tantamount to cosmic order and this seems to be the main lesson taught by the philosopher of *panta rhei*.

---

[7] First philosophical use of the word *aion* in the sense of eternity is detected in Empedocles (31B16) by A[ndré] J. Festugière, Le sens philosophique du mot αἰών, *La parola del passato* 11 (1949), 176.

[8] É[mile] Benveniste, Expression indo-européenne de l'"éternité," *Bulletin de la société de linguistique de Paris* 38 (1937), 104, 107; Enzo Degani, *Αἰών*, Bologna: Pàtron Editore 2001 [1961], 11-12, 15-16.

[9] Marcel Conche, commentaries in Héraclite, *Fragments*, Paris: PUF 1986, 446-447 with references to Homer and Pindar.

[10] It is contended that *aei* possesses the meaning of eternity already in Homer (cf. ever-living gods) and *aion* acquired its abstract philosophical meaning of eternity as nominal transposition of *aei*, Benveniste, 110.

[11] "Aion is the eternal, the divine, which enters into the temporal [realm and into] human life," Günter Wohlfart, *"Also sprach Herakleitos": Heraklits Fragment B52 und Nietzsches Heraklit-Rezeption*, Freiburg: Karl Alber 1991, 53; the word *aion* in B52 refers at the same time to the eternal divine being – cf. Homer's ever-living gods – and to the time of human life; *aion* is understood as something renewing itself constantly in time, pp. 50-51.

[12] *Pessoi* was a board game known in two forms, *polis*, a game of skills, played with many pieces without dice, and *pente grammai* played with pieces and very likely also with dice, Roland G. Austin, Greek board-games, *Antiquity* 14 (1940), 257-271; Leslie Kurke, Ancient Greek board games and how to play them, *Classical Philology* 94 (1999), 253-257. The fixed character of the game rules in interpretation of B52 is emphasized by Kahn, *op. cit.*, 227.

[13] Cf. Robinson, *op. cit.*, 117.

5 PARMENIDES AND MELISSUS

Parmenides was the first to undertake a task of building a fully monistic philosophical system. In a poem in which he announced truths divinely revealed to him, he described Being as one and indivisible (28B8.22-25, B4), timeless (B8.5-6, 19-20), ungenerated and imperishable (B8.6-21), changeless, immovable, and complete (B8.26-33). The extant fragments indicate that there is a fundamental agreement between Parmenides and Melissus, concerning characteristics of Being. Like Parmenides, Melissus asserted that Being is (30B1, B2, B4), immovable (B7.7-10, B10), complete (B2), and unique (B5, B6). The physical world is unreal because it is characterized by "change, multiplicity, temporal succession and imperfection" (B8). Being cannot be known through sensory perception because senses indicate that things are constantly changing, which directly contradicts the immutability of Being (B7). However, as commonly assumed, there is at least one fundamental difference between them. Melissus considered Being infinite, whereas for Parmenides Being was finite because it was held in limits (28B8.26, 31, 42) and was compared to a sphere (B8.42-43). Does the limited/unlimited difference signify the modification introduced by Melissus to the Eleatic philosophy?

5.1 UNLIMITED AND INFINITE

It is sometimes asserted that the term πεῖρας, used by Parmenides, and ἄπειρον, used by Melissus, refer to the same attribute of Being.[1] This view may be attributed to the fact that Parmenides never used the term *apeiron*[2]; therefore, to contrast his *peiras* with the *apeiron*, the supporters of this view take the lacking *apeiron* from Melissus, which is understandable because both Parmenides and Melissus represent the same philosophical outlook. So it appears. However, an interpretation can be defended that the two thinkers express two different attributes.

Limits are used as an argument for completeness of Being. Parmenides said that Ananke holds Being in the bonds of a limit "since it is not right for the Being to be incomplete (οὐκ ἀτελεύτητον)" (B8.30-32) and "because there is an outermost/ultimate limit, it is completed (τετελεσμένον)" (B8.42). In these statements

---

[1] The fact that Being is without beginning and without end was expressed by Parmenides through *peiras* and by Melissus through *apeiron*, Joseph Owens, *A history of Ancient Western philosophy*, New York: Appleton-Century-Crofts 1959, 92-93, 95, 97. Melissus meant by *apeiron* what Parmenides meant by *peiras*: "that which must be undifferentiated" both spatially and temporally, Leonardo Tarán, *Parmenides*, Princeton: Princeton University Press 1965, 153-154, 158, 200-201. The Eleatics were using terms *peiras* and *apeiron* to express the fact that Being is motionless, everlasting, unique, homogenous, full, perfect, and complete, Leo Sweeney, *Infinity in Presocratics: a bibliographical and philosophical study*, The Hague: Martinus Nijhoff 1972, 134.

[2] Melissus uses both terms, cf. B6.

the existence of limit is used twice as an argument that Being possesses τέλος, that is, Being is full or complete.[3] However, it is possible to have more than one complete Being, therefore, Parmenides' argument is, technically, incorrect because it may be argued that the limitedness of Being, even a complete Being, leaves room for the possibility that there are many complete Beings.[4] And this was important for Melissus in whose fragments there is a new tone which is absent in Parmenides, namely an emphasis on the fact that Being is one. Parmenides stated explicitly (B8.6) and implicitly (B8.22-25) that Being is one, but the oneness of Being is much more prominent in Melissus and is used as an argument for other attributes of Being. The new emphasis is most likely the reason for stating that Being is infinite. In this situation, the question of what can possibly be outside limits seems to disappear and with it a suspicion that there may be multiple beings.[5]

However, with the new emphasis, Melissus did not introduce a contradiction into Eleatic ontology by stating that Being is finite and infinite. The avoidance of a contradiction in the statement that Being possesses both a *peiras*-attribute and *apeiron*-attribute is possible when we realize that *peiras* used by Parmenides is not an opposite of the *apeiron* discussed by Melissus. We have to make a distinction between two meanings of *peiras* and *apeiron* and, consequently, between two pairs of opposites. On the one hand, we have the pair *peiras*$_1$-*apeiron*$_1$ or limited-unlimited. On the other hand, the pair *peiras*$_2$-*apeiron*$_2$ or finite-infinite.[6] Parmenides is writing about *peiras*$_1$, that is, about the fact that Being is within limits. Melissus told us that Being is *apeiron*$_2$, that is, infinite. There is no contradiction between them, there is only a different emphasis.

It is important that although Parmenides referred to the *peiras*$_1$-attribute, he took it metaphorically, not literally. Being is perfect, and as perfect, it is in the bonds of *peiras*, that is, not surrounded by a spatio-temporal boundary, but it is determined, defined. Because Being surpasses the boundaries of time and space, because Being is incorporeal, *peiras* is not a physical limit which demarcates it from something else, because if it had been, then an entity outside the limit would have to be another Being – and Being is only one, or it would be nonbeing – but nonbeing does not exists.

---

[3] Jaap Mansfeld, *Die Offenbarung des Parmenides und die menschliche Welt*, Assen: Van Gorcum 1964, 100; Alexander P.D. Mourelatos, *The route of Parmenides*, New Haven: Yale University Press 1970, 121.

[4] It was clear for Aristotle that there may exist many complete entities (*Met.* 1021b12-1022a3, 1055a10-18), hence completeness does not entail oneness.

[5] According to Tannery, who considered Melissus' One to be incorporeal, for Melissus, infinity "is just the negation of existence of other beings; thus, it is only a synonym of the totality for the ensemble of phenomena," Paul Tannery, *Pour l'histoire de la science hellène de Thales á Empédocle*, Paris: Alcan 1887, 266.

[6] Sinnige said that beginning with the Pythagoreans, *apeiron* means infinite in a positive sense or infinite in the negative sense, the latter being synonymous with or undeterminedness, so that *peiras* means determinedness or well-determinedness, Theo G. Sinnige, *Matter and infinity in the Presocratic schools and Plato*, Assen: Van Gorcum 1968, 44-47.

5.2 SPHERE

If limits discussed by Parmenides are considered metaphorically, then so should the much discussed sphere simile. Parmenides said that "because there is an outermost limit, it [Being] is completed, in every direction, similar to the mass of a well-rounded sphere, from the center everywhere equally balanced; for it must not be either greater or smaller here or there" (B8.42-45). It is clear that, according to Parmenides, Being is not spherical, but that it is like a sphere or like the mass of a sphere. It is not certain, however, what exactly is the point of the simile. But in most proposed interpretations one theme is common: perfection. For example, the view that the sphere simile points to "ausgeglichene Kräftespiel der Gewichtsmasse"[7] or to the fact that "the mass of a sphere in equilibrium around the middle is in all parts of equal strength,"[8] can be considered to mean perfect balance[9] or perfect distribution of force around one point. The comparison of Being with a sphere stresses that "Being is undifferentiated" continuity[10] and points to perfect undifferentiation, perfect uniformity, and perfect continuity (continuity is perfect or there is no continuity at all). Therefore, it seems that the simplest explanation is that Parmenides' "image of the sphere represents Being as a self-determining *entelechy* or perfection."[11] In this, Parmenides used an image which has been used all along as a shape of the universe or present in the universe. Even before Anaximander, the universe was thought to be spherical or the shape of an egg with a solid vault of heavens above and the nether world of Tartarus below with a disk-shaped earth in between. A spherical universe is also in Brahmin cosmography (*Rigveda* 10.44.8, 3.55.20, *Atharvaveda* 20.94.8). Similar ideas are known to primitive tribes.[12]

For Anaximander the spherical shape of the universe was used as an argument supporting the claim that "the earth is on high, held up by nothing, but remaining on account of its similar distance from all things," that is, the cylindrical earth is suspended in the center of the universe (12A11). According to doxographic tradition, Xenophanes' God was spherical, for Empedocles the *Sphairos* was inherently spherical.

The spherical attribute was used also after Parmenides. Plato's Demiurge eventually shaped the prime matter into spherical earth, Aristotle's universe was filled with fifty-five concentric spheres.[13] Spherical shape has always been considered a perfect shape for the universe, and in that respect Parmenides did not break with tradition. His sphere simile simply expressed the conviction that Being is perfect; as true reality, the only reality, with divine attributes, it cannot be otherwise, and to stress the fact of this perfection Parmenides used the simile which traditionally has been associated with perfection of objective reality. This simile was Parmenides' bow toward traditional world imagery, as though Parmenides wanted to say that at least in this one respect the views before him were correct, namely in seeing reality if not as perfect, then at least as a reflection of perfection.

With the sphere imagery Parmenides explicitly maintained ties with the views espoused before him, and, importantly, he seemed to retain this imagery in the way of opinion by presenting a cosmography in terms of rings present in the universe (B12, A37). If the rings are the same as spheres, then the way of opinion can be considered as the traditional world image which is not altogether false. This world image is due to sensory cognition and as such, faulty. Truth can be obtained only through non-sensory means (and only through the assistance of the goddess from B1) as presented in the way of truth. In Parmenides' eyes, his predecessors did not reach that level and thus were incapable of knowing the truth the way it is. But on at least one point they were correct, namely in seeing the world as framed by the image of a sphere.

5.3 PHILOSOPHICAL ENVIRONMENT

There arise two questions. First, why is the *peiras*$_1$-attribute so important to Parmenides? Second, why is the *apeiron*$_2$-attribute so important to Melissus? The answers may be found in the two different intellectual environments in which the two Eleatic philosophers developed their ideas. For Parmenides it was the Pythagoreanism which permeated the intellectual atmosphere of his environment; for Melissus it was the Milesians.

Ontological and mystical importance of the number was the reason why the concept of *apeiron* acquired a negative value in Pythagoreanism. All numbers are finite, all ratios between numbers are finite; therefore, the *apeiron* undermines the perfection of musical scale and celestial harmony, and thus it should be avoided. This negative status of the *apeiron*, so prominently espoused later in Aristotle, was accepted by Parmenides. From the *peiras-apeiron* pair which begins the Pythagorean table of opposites, Parmenides retained only *peiras*. Being is the only true reality, and it does not admit of negative attributes; therefore, it cannot be designated with the Pythagorean *apeiron*-attribute.

Melissus did not feel confined by the Pythagorean valuation of the *apeiron* and returned to the positive understanding of the *apeiron*, which term has in Milesian philosophy, in particular, in Anaximander's system.[14] This is due to Melissus' "more immediate Ionian surroundings,"[15] that is, to his living in Samos, in proximity to Ionia and thus Miletus, unlike Parmenides who lived in Elea, in southern Italy, and this region was the home of Pythagoreanism. Not incidentally, Parmenides associated himself with the Pythagorean Ameinias and even built a statue of him after his death (A1, A12).

---

[7] Hermann Fränkel, *Wege und Formen frügriechischen Denkens*, München: Beck 1955, 196.

[8] Tarán, *op. cit.*, 158.

[9] The sphere simile says that the subject's "uniformity is like the perfect balance of a ball about its centre," G.E.L. Owen, Eleatic questions, in R.E. Allen, D.J. Furley (eds.), *Studies in Presocratic philosophy*, Atlantic Highlands: Humanities Press 1975, v. 2, 66.

[10] Tarán, *op. cit.*, 158, 155.

[11] A.H. Coxon, *The fragments of Parmenides*, Assen: Van Gorcum 1986, p. 217; Mourelatos, *op. cit.*, 126; Jerzy Pawliszcze, Parmenidesa kula bytu: fragm. B8.43-45, *Analiza i Egzystencja* 4 (2006), 19.

[12] Fritz Graebner, *Das Weltbild der Primitiven*, München: Reinhardt 1924, 119.

[13] Dietrich Mahnke, *Unendliche Sphäre und Allmittelpunkt*, Halle 1937 [reprint, Stuttgart: Frommann-Holzboog 1966], ch. 5.

[14] Melissus is an "avowed follower of Anaximander" and "represents the renascence of the Milesian philosophy on nature, which becomes prominent about the middle of the fifth century," Werner Jaeger, *The theology of the early Greek philosophers*, Oxford: Clarendon Press 1947, 27, 28.

Melissus may have chosen to concentrate on the *apeiron* in his discussion of Being because of Zeno's paradoxes.[16] The paradoxes were due to the *apeiron*-property of the *peiras*-entity, i.e., to the infinitistic character of a limited entity. To defuse a possibility of such a paradox within Being, Melissus abandoned the discussion of *peiras* altogether.

Another reason for Melissus' concentration on *apeiron* is a possible criticism of Parmenides' sphere simile.[17] The use of this simile certainly invokes an image of a limited Being, which was hardly the intention of Parmenides.

Parmenides did not renounce an infinitistic vision of Being. He did not call it *apeiron*, but he still thought it was infinite, which we see in his use of ἀτέλεστον (B8.4).[18] This is a much disputed term, and it is sometimes translated "not accomplished" or "imperfect," which would immediately contradict the designation of Being as οὐκ ἀτελεύτητον, not incomplete (B8.32), and τετελεσμένον, completed (B8.42). Therefore, very often different emendations are proposed to resolve this contradiction.[19] However, these emendations may not be necessary if two different meanings of the verb τελέω are taken into account, namely, to complete, accomplish, fulfill, and to finish, end. Using these two meanings, ἀτελεύτητον and τετελεσμένον can retain the meanings just used, and ἀτέλεστον can be translated as without end, endless, in-*finable* (to use the old French word *finable*), or infinite.[20] This interpretation brings back consistency to Parmenides' poem and allows us to see not only that completion or perfection of the Eleatic Being was seen by Parmenides as consonant with its infinity, but also that his discussion of limits did not eliminate infinity from Being. The latter case can be strengthened by indicating other tacit references to infinity.

In his discussion of the attributes of Being, Parmenides said that it is ἄναρχον and ἄπαυστον (B8.27). The word ἀρχή in Homer and Hesiod referred to the initiation of an action[21]; therefore, *anarchon* can be translated as without beginning, without start, that is, with respect to its origin, Being has no limit, no boundary which can be identified as its starting point. However, to avoid a contradiction with the statement that Being does have limits held by Dike and Ananke, it is better to see in *anarchon* an expression of the conviction that with respect to duration, Being is infinite. The same can be stated about *apauston*. The word stems from the adjective παυστός, which is nowhere attested and as such is coined by Parmenides from παύω, to stop, to bring to an end. Consequently, *apauston* means without end, without pause, that is, infinite in respect to the future duration. Being is *anarchon* and *apauston*; its duration is infinite. We may agree, therefore, with the statement that these two designations "echo Anaximander's account of the Infinite."[22]

This discussion seems to indicate that Parmenides was not indifferent to Anaximander's theory, and he saw infinity as compatible with his own ontological account. However, he was more concerned about the perfection of Being as seen from the Pythagorean perspective. Being is perfect because it is determined. The determinacy of Being overshadows its infinity. Anaximander would see perfection of reality within the framework of infinity; Parmenides saw infinity within the framework of perfection. Because the latter was expressed through *peiras*, Parmenides' use of this term was taken to mean that *apeiron* can in no wise be compatible with Being.[23]

It is important to emphasize that by allowing the presence of *apeiron* in his theory, Parmenides is not using a non-Pythagorean element. Pythagorean philosophy also refers to *apeiron*, so that Pythagoras is even considered to have been influenced in that respect by Anaximander.[24]

Like Pythagoreans, Parmenides deemphasized the role of the *apeiron*, but did not remove its presence from his theory. According to the Pythagoreans, before the beginning of the world there was the *apeiron* and *peras*. There is nothing that precedes Parmenides' Being, therefore the *apeiron* attribute is integrated with Being, and it is its characteristic which does not undermine Being's perfection because Being has limits in the sense of being determined. It does not require any determinacy which is separate from it like Pythagorean *peras* is separate from *apeiron*, but the infinite Being is its own determinacy; infinity is an attribute of Being whose other attribute is that it is well-determined. Therefore, when Melissus explicitly discussed the *apeiron*-attribute, he was not making innovations in the Eleatic ontology, but he was bringing to light what has been concealed in Parmenides all along.[25]

---

[15] Owens, *op. cit.*, 93

[16] Sweeney, *op. cit.*, 132-134.

[17] Sweeney, *op. cit.*, 132-134.

[18] Already Simplicius indicated this meaning of ἀτέλεστον and rightly pointed to Melissus' *apeiron* (*In Phys*. 29.26-30).

[19] The phrase ἠδ' ἀτέλεστον is emended, for example, to οὐδ' ἀτέλεστον, nor incomplete, to ἠδὲ τελεστόν, and complete, or to ἠδὲ τελεῖον (or ἠδὲ τέλῆεν), and complete, Mourelatos, *op. cit.*, 96; Owen, *op. cit.*, 77; P.A. Meijer, *Parmenides beyond the gates*, Amsterdam: Gieben 1997, 27 note 166. The emendations aim at supplying the second negation, οὐ, or at removing the existing negation ἀ-.

[20] Lindell-Scott-Jones dictionary; see also Hermann Diels, *Parmenides Lehrgedicht*, Berlin: Reimer 1897, 75; Sinnige, *op. cit.*, 46; Denis O'Brien's commentary in P. Aubenque (ed.), *Études sur Parménide*, Paris: Vrin 1987, v. 1, 48; Pierre Aubenque, Syntax et sémantique de l'être dans le poèm de Parmènide, in Aubenque (ed.), *op. cit.*, v. 2, 106.

[21] Mourelatos, *op. cit.*, 116.

[22] Coxon, *op. cit.*, 206.

[23] Parmenides is even supposed to have been "the apostle of a finite Being," Scott Austin, *Parmenides: Being, bounds, and logic*, New Haven: Yale University Press, 139.

[24] James A. Philip, *Pythagoras and early Pythagoreanism*, Toronto: University of Toronto Press 1966, 69.

[25] Thereby it is not true that Melissus gave the Eleatic Being "the new attribute of infinity," N.B. Booth, Did Melissus believe in incorporeal Being?, *American Journal of Philology* 79 (1958), 63.

5.4 INCORPOREALITY OF BEING

There still remains one question. "Incorporeality of Being was never asserted by Parmenides, although his whole system cries out for such a doctrine; this we do find in Melissus."[26] But, on the one hand, Melissus very explicitly stated that Being was "infinite in magnitude" (μέγεθος, B3); on the other hand, Being had no body (σῶμα), no solidity (πάχος, B9). Is Being really incorporeal? And if it is, how can an incorporeal entity have a magnitude, let alone be infinite in magnitude?

One answer is that "the infinity in question is that of beginningless and endless duration, not that of unlimited spatial extension. The former could be asserted by Melissus of incorporeal Being, while the latter obviously could not."[27] In this way, magnitude has only temporal character and incorporeality means having no extension. This is true incorporeality in the Cartesian sense, but it appears that this explanation does no go far enough. It is conceivable that for Melissus having no body and no solidity meant not only having no extension but also being atemporal. In this interpretation, the word magnitude is emptied of any physical meaning and is taken purely figuratively.

It is testified that the word magnitude may have a metaphoric meaning. Already Homer frequently used words like μέγας metaphorically. Melissus himself said about the greatest proof, μέγιστον σημεῖον (B8.1).[28] When Melissus said that Being cannot become larger (μεῖζον), he did not mean a spatial change, but the change in respect to greatness or perfection.[29] After all, when Xenophanes said that "God is greatest" (B23), greatness could hardly be understood here spatially. Therefore, infinity in magnitude implied by Being's existence in B3 can be understood as Being that is "existentially without limit" because it is one, unique, all-inclusive.[30]

However, the way Melissus discussed infinity of Being seems to imply that it is not purely incorporeal infinity because he said that Being "always was and always will be and it has no beginning nor end but is infinite" (B2). So it appears that Being has past and future. But when Melissus said that Being has no beginning nor end but is *apeiron*, then this may be taken to mean that Being's existence is independent of temporal constraints to the extent that it exists outside the temporal boundaries. Being is extratemporal, it exists outside of time, and thus its duration is infinite, where duration is not temporal, but extratemporal, eternal present, duration proper. However, because we think about Being in spatio-temporal terms, its infinity is also expressed in these terms, namely, that "it always was and always will be and it has no beginning nor end." If the temporal framework is taken into account, then, to be sure, Being always existed and always will exist, but this does not mean that the framework is inextricably connected with it. The spatio-temporal framework is an inherent part of the way of opinion, it appears to be indispensable for sensory cognition and through this framework Being manifests itself to mortals in an inherently distorted form as changeable, divisible, etc. But still, Being so filtered through this framework manifests itself as having no beginning nor end; the whole of the phenomenal reality has no beginning nor end. Therefore, when in B2 Melissus was discussing the fact that Being is *apeiron*, he seems to have meant that its existence or duration is infinite, that it is in the perennial, atemporarily ceaseless now.[31]

To sum up: A confusion concerning the use of *peiras* and *apeiron* by the Eleatics stems from the fact that Melissus did not put himself in opposition to Parmenides with his discussion of *apeiron*. Melissus silently abandoned the problem of *peiras*$_1$ so perspicuous in Parmenides' poem and discussed the *apeiron*$_2$-attribute of Being to which Parmenides alluded only indirectly in his poem. Melissus, in a sense, became "more Parmenidean than Parmenides" but not by choosing infinity over the finite, because the finite cast a shadow on the incorporeal character of Being,[32] but by stressing the former over the latter. He became also more Parmenidean than Parmenides by emphasizing the incorporeal character of Being, which is only implicit in Parmenides. This required treating the terms *peiras* and *apeiron* used by the Eleatics only metaphorically, which only aggravated the problem of resolving an apparent contradiction between them. But it seems that the contradiction can be resolved satisfactorily.

---

[26] Gregory Vlastos, Zeno of Elea, in *Encyclopedia of Philosophy*, v. 8, 377.

[27] Gregory Vlastos, Raven's 'Pythagoreans and Eleatics,' in Allen, Furley, *op. cit.*, v. 2, 173.

[28] Johannes H. Loenen, *Parmenides, Melissus, Gorgias*, Assen: Van Gorcum 1959, 149 note 41.

[29] Loenen, *op. cit.*, 157-158.

[30] Andreas Graeser, *The argument of Melissus*, Athens: Hellenic Society for Humanistic Studies 1972, 15.

[31] Or "permanent present," Néstor-Luis Cordero, *By being, it is: the thesis of Parmenides*, Las Vegas: Parmenides Publishing 2004, 171. Cf. Tarán, *op. cit.*, 180 note 2.

[32] Barbara Cassin, *Si Parménide*, Lille: Presses Universitaires de Lille 1980, 33. But Cassin also said that "we should not oppose the unlimited of Melissus … to the limited sphere of Parmenides," which is an Aristotelian interpretation, p. 143. That the limited character of Being does not contradict its unlimited character was also admitted by Hans von Steuben, Wahrheit und Gesetz. Die Offenbarung des Parmenides, in Parmenides, *Über das Sein*, Stuttgart: Reclam 1981, 109, 111-112, 116, 124-125, 128-129.

6 ZENO'S PARADOXES

A grand but paradoxical vision of Being offered by Parmenides had its detractors and Zeno set about to defend the Eleatic ontology by showing that the views of the adversaries lead to contradictory, therefore unacceptable, conclusions. Zeno presented two types of arguments. The arguments against plurality were designed in support of Parmenides' doctrine that reality is one and indivisible, not many, not a plurality. Parmenides also believed that reality is motionless, and the second type of arguments were designed to show impossibility of motion. The four *aporiai* against motion were reported by Aristotle (*Phys.* 6.9) and they may be today the most widely known arguments of ancient Greek philosophy. The common assumption of both types of arguments is infinite divisibility of time and space, i.e., paradoxes of time and space are at the heart paradoxes of infinity.

Among the arguments against motion as reported by Aristotle, only one argument (the dichotomy) explicitly states that it is against motion. However, that much was clear for Aristotle from his many allusions [1].

6.1 THE DICHOTOMY

"The first [argument] about the non-existence of motion is based on that something moving must arrive earlier at the half [point] than at the end [of the way]" (*Phys.* 239b11-13 = 29A25). The statement is so succinct that the essence of the argument can hardly be recognized. However, Aristotle discussed the argument earlier and later in the text,[2] which is somewhat helpful in unraveling its meaning. The point of the argument is that if something is moving from A to B, it has to arrive at the mid-point C between A and B. However, before arriving to C, it has to reach the mid-point D between A and C. Such bisections can be continued infinitely so that, according to the argument, it is impossible to begin to move. This is because there are infinitely many points A … D C B which have to be traversed to reach B, and infinitely many points cannot be traversed in finite time. Also, there is no next point to which one can move from A. Hence, no motion. Because this is to be an argument against the existence of motion, this version – that can be called regressive version of the dichotomy argument – seems to be original. However, based on Aristotle's discussions, another, progressive version is also possible[3]: to arrive from A to B, we have to reach the mid-point C, after which we have to reach the mid-point D between C and B, so that infinitely many points A C D … B have to be traversed before reaching the end-point B. This cannot be done in finite time, therefore, the conclusion would be that we are constantly in motion without a prospect of reaching the goal.[4] What is common to both variants is the fact that the end-point B can never be reached, i.e., never in finite time because our intuition seems to indicate that it is not possible to process (in particular, traverse) infinity in finite time. Therefore, this clash of infinity with the finite seems to point to the illusory aspect of processing itself. Incidentally, this kind of reasoning opens a prospect of immortality: to reach age B starting from age A, we would have to reach age C, a half-way between A and B. By continuing such bisections, we could conclude that we do not age at all.[5]

The argument that space is infinitely divisible is an unspoken assumption. In the universe in which the smallest distance exists (granular space), the argument could not be true, since the bisection process would have to stop at a certain point. To assure infinite divisibility, it is sufficient that space is everywhere dense, and not even continuous. In fact, for a particular interval, it is enough that it is dense about one point x, i.e., that there is a point in the interval between any other point in the interval and x. However, because the dichotomy could also be applied to any subinterval of a particular interval, the interval has to be everywhere dense. The density is necessary to fit an infinity of points into a finite interval. Consequently, there is a gap inside the interval where there are no two immediately neighboring points: in the regressive version, no next point immediately after A, in the progressive version, no point immediately before B. The focus of the argument is not the existence of such a gap, but its reason, the infinity of points crammed into finite interval. To see that better, Aristotle elaborated on the argument by requiring to count all the mid-points. This is deemed impossible, since there is no number (no natural number, that is) which would express the infinite number of such mid-points (*Phys.* 263a4-11). This version, however, is not Zeno's; some unspecified people expressed it in this form (a7).

The problem is sometimes solved by stating that the sum of intervals into which the interval [A, B] is split is finite, i.e., assuming that B – A = 1, $\sum_{i=1}^{\infty}\frac{1}{2^i}=1$, which is a redundant statement at best since from the beginning it is assumed that [A, B] is of finite length and the bisection process creates only nonoverlapping subintervals of [A, B], i.e., it is known from the start that $1=\sum_{i=1}^{\infty}\frac{1}{2^i}$. Moreover, associating infinite divisibility of space with infinite divisibility of time may not be helpful because the argument can also be rephrased in temporal terms (we do not age), and thus we apparently cannot reach point B in space just as we cannot reach an instant to which B is mapped. The problem is not that we stand still – in illusory fashion or otherwise, we do reach B – but how it is that an infinity of steps, or, in general, an infinity of tasks (super-tasks) can be reconciled with finite time. A surprisingly large number of attempts made over two and a half millennia to solve this quandary testifies to the fact that the problem is far from trivial.

---

[1] *Anal. pr.* 65b18, *De sophisticis elenchis* 179b20, *Top.* 160b7, cf. the beliefs of the ancient commentators on the *Physics*, Philoponus 802.31, Themistius 186.30, Simplicius 1012.35.

[2] *Phys.* 233a21-31, 263a4-b8, also, *De lineis insecabilibus* 968a19-24.

[3] Regressive and progressive versions, Gregory Vlastos, Zeno's race course, *Journal of the History of Philosophy* 5 (1966), 95; J[ohn] A. Faris, *The paradoxes of Zeno*, Aldershot: Avebury 1996, 11-12.

[4] Actually, the second version can be extended to provide the conclusion given by the first version: the second version is applied to the interval [A, B], but it can also be applied to any subinterval [A, C], however small, i.e., we cannot move from A, cf. H.D.P. Lee, *Zeno of Elea*, London: Cambridge University Press 1936 [reprint, Amsterdam: Hakkert 1967], 71.

[5] F[azal] A. Shamsi, *Towards a definitive solution of Zeno's paradoxes*, Karachi: The Times Press 1973, 29; Rafael Ferber, *Zenons Paradoxien der Bewegung und die Struktur von Raum und Zeit*, München: Beck 1981, 7.

6.2 THE ACHILLES

"The second [argument] is the so-called Achilles: it states that the slowest runner [= tortoise] can never be overtaken by the swiftest [= Achilles]; for the pursuer must first reach the point, from which the pursued started, so that the slower must always be ahead" (Aristotle, *Phys.* 239b24-28 = A26). As properly observed by Aristotle, there is little conceptual difference between the dichotomy (progressive version) and the Achilles. In the former, distances are bisected; in the latter, the distance under consideration at a particular stage is the distance between the starting point for Achilles and the starting point of the tortoise. That is, when Achilles starts at point A and the tortoise at point C, then by the time Achilles reaches C, the tortoise is at point D, and by the time Achilles reaches D, the tortoise is at E, etc., always ahead of Achilles. They cannot reach a meeting point B, because such a point would follow an infinity of points A C D E … B, and such an infinity of points cannot be covered in finite time. To see that, in fact, the Achilles can be reduced to the progressive version of the dichotomy, assume that Achilles runs twice the speed of the tortoise and that C, the starting point of the tortoise, is half way between A and B. Then the tortoise reaches the mid-point between C and B at the time when Achilles reaches C, etc. The walk of the tortoise from C to B is also the repetition of the dichotomy. That is, the Achilles is two dichotomy arguments for the situation when Achilles is in lockstep with the tortoise and is always one subinterval behind the tortoise. Therefore, there is a one-to-one correspondence between steps made by Achilles and the tortoise with one subinterval off between the two sequences of steps.

6.3 THE FLYING ARROW

The dichotomy and the Achilles can be solved by saying that there is a one-to-one correspondence between the points of the interval and the time-points between the moments of start and finish. Time is infinitely divisible in the same way that space is. Thus, in finite time finite, space can be covered in spite of this infinite divisibility. Yet another argument is offered to address this apparent solution.

The third argument states that "the flying arrow is at rest, which results from the assumption that time is composed of moments/nows; if this assumption is not granted, the conclusion will not follow" (*Phys.* 239b30-32 = A27). More precisely, "if, he says, everything always is at rest when it occupies an equal space and that which is in motion is always in a now, the flying arrow is motionless" (239b5-7 = A27).[6]

This succinct statement says explicitly that when the arrow is occupying space equal to the arrow itself, it is at rest, and that the flying arrow is always in a particular instant. Afterwards, an implicit assumption is made: in each instant/now, the arrow occupies space that is equal to the arrow, hence, the flying arrow does not really fly – a contradiction, an absurdity.

First, a now, or an instant, should be understood as a point, not as an atomic interval of time. This is the way Aristotle understood a now[7] and, as it seems, Zeno did as well: in the dichotomy and in the Achilles, infinite divisibility of space was assumed, and it appears that infinite divisibility of time was just as obvious to Zeno. Second, the paradox can be disabled by invalidating the assumption that whatever occupies a space equal to itself is at rest. To determine that something is in motion or at rest, positions of an object in two different moments should be compared. When looking at a snapshot of an arrow, it is impossible to determine whether it is at rest or in motion. Our judgment in that situation should simply be suspended. The arrow does move in a particular instant or does rest, it is just that we cannot determine it when looking at its position in one point-instant. The determination can be made when a context is known, i.e., positions before or after the instant in question. It is thus not true that in an instant, the arrow is neither at rest nor in motion, as claimed by Aristotle (*Phys.* 239b1-2).[8] Actually, it is not always necessary to suspend judgment in such a situation. Consider a snapshot – or a vase painting – of a runner. It would be difficult to deny that in the picture, the runner is in motion. A context is not set aside here, since the interpretation of the picture calls for our knowledge of how one runs and how one rests; this background knowledge tells us that a runner with his legs extended in midair is not at rest, or that it is possible to say with some certainty what he is doing.[9]

---

[6] The text seems to be corrupted and requires some emendations based on commentaries of Simplicius, Philoponus, and Themistius, see Lee, *op. cit.* 78-81; Maurice Caveing, *Zénon d'Élée: prolégomènes aux doctrines du continu. Étude historique et critique des fragments et témoignages*, Paris: Vrin 1982, 96-98.

[7] E.g., *Phys.* 219a29, 222a10-12, 17-19, 233b33-234a3, and argument against atomic time in 263b26-264a6; F. R[odney] Pickering, Aristotle on Zeno and the now, *Phronesis* 23 (1978), 253-257; Caveing, *op. cit.*, 102-105; Faris, *op. cit.*, 37-42.

[8] Followed by V.C. Chappell, Time and Zeno's arrow, *Journal of Philosophy* 59 (1962), 203, Gregory Vlastos, A note on Zeno's arrow, *Phronesis* 11 (1966), 13-16, and Caveing, *op. cit.*, 101, who agreed with Vlastos. "A thing should be in space without being in motion and without being at rest. True, this unthinkable state should be only for an imaginary instant; but still, how does it help? We believe that we cannot dispose of an instant and, at the same time, we see that this concept is so vacuous as an empty shell and we cannot think about an instantaneous state without contradictions, which are mockery to the law of the excluded middle, and thus we did not gain anything save the realization that there is something [left] unexplained," Eugen Raab, *Die zenonischen Beweise*, Schweinfurt: Reichardt 1880, 41. It appears that, unawares, Aristotle himself made use of motion at an instant, David Bostock, Aristotle on continuity in *Physics* vi, in L. Judson (ed.), *Aristotle's* Physics: *a collection of essays*, Oxford: Clarendon Press 1991, 195. The necessity of considering the context is stressed by David Konstan, Problems in Epicurean physics, *Isis* 70 (1979), 396.

If an interval is composed of instants, and the arrow moves within an interval, then consequently, it also moves in instants constituting this interval although it appears stationary when looking only at one instant. Zeno's argument is made in the wrong direction. The argument says that what is true about instants is also true about an entire interval, and because the assumption was made that the arrow is at rest in an instant, so it is for the interval. But reasoning should begin with the interval: what is true for the entire interval is true for all instants, at least in the matter of motion or rest.[10]

6.4 THE STADIUM

"The fourth [argument] is about equal bodies moving alongside/opposite equal [bodies] in the stadium, the ones from the end/goal of the stadium, the others from the middle, with equal speed, in which he thinks it follows that half the time is equal to its double. The paralogism is in requiring that a body moving at an equal speed takes an equal time to pass a moving body and a body of the same size at rest" (*Phys.* 239b33-240a4 = A28). Aristotle then describes a situation illustrated by Diels from Simplicius' report of Alexander's account (A28):

A A A A
B B B B →
← C C C C

where the As are stationary and Bs and Cs move with equal speed in the indicated directions. When the original positions are as in the diagram, then after the rightmost B is aligned with the rightmost A, this B passes two As. At the same time, this B passes four Cs to become aligned with the rightmost C, as in

A A A A
B B B B →
← C C C C

i.e., the rightmost B passed twice as many Cs as As. Aristotle ascribed to Zeno an assumption that it always takes B the same time to pass any body A or C of equal size regardless of its being in motion or at rest. But with the discussed setup, it appears that it takes half the unit of time to pass C and the whole unit to pass A, i.e., "half the time is equal to its double," an absurdity.

The root of the problem appears to be in the assumption that it takes the same time to pass a body at rest as it does to pass the same body in motion. A widespread sentiment against this assumption is expressed by the statement that "this is either a manifest absurdity, so manifest, in fact, that we have the right to ask whether Zeno, in proposing this difficulty, was speaking seriously or whether he only tried, as it was frequently the case in Greece, to trap his adversaries in a plausible but absurd sophism."[11] It is worth noticing, however, that today's science lives under the shadow of such an assumption. A cornerstone of the theory of relativity is the idea that the speed of light is constant in respect to any frame of reference, any observer, stationary or otherwise. However, the assumption is very unintuitive and the consequences are still more unintuitive.

Because the assumption made by Zeno was considered counterintuitive at best and absurd at worst, there were numerous attempts to rationalize it.

One way of rationalizing Zeno's assumption is to state that "Aristotle has entirely missed the point" by not noticing that Zeno worked with the assumption of atomic time and space.[12] Because no action can be performed in half an instant, some form of Zeno's assumption can be derived from the granularity of time – a body can pass another body in one infinitesimal time unit regardless of the relative speed of the objects. Also, assuming that As, Bs, and Cs are of the smallest size (but why would a stadium be needed to illustrate this?), if A passes a B in one time unit, it would have to pass an A in half a unit – a contradiction.

Another way of defending Zeno's assumption is by stating that it refers not to passing, but to standing against, i.e., not to the entire moment when two bodies pass one another, but when two bodies (of equal size) are exactly aligned with one another.[13] During an instant, such two bodies are aligned and, in fact, one could not even say whether they are in motion or at rest – a lesson from the arrow argument. However, if instants are considered dimensionless, for an instant when a B is aligned with one A, there will be two instances when the same B is aligned with two consecutive Cs. Two dimensionless instances last for as long as one instance, whereby the paradox disappears. If instances are considered to be atomic, this approach is reduced to the previous one.

Another way of approaching the problem is by excising some parts of Aristotle's report as his addition or even misrepresentation of Zeno's original statement. In one such attempt only two bodies are retained, one moving throughout the entire stadium, one only through its half. The number of instants needed to accomplish the task is infinite in both cases, and the infinites are equal, i.e., it takes the same time for one body to move by distance x and for the body to move the distance $2x$.[14] The interpretation, interesting as it is since it focuses on the infinity as the source of the paradox, is too invasive because it changes Aristotle's report almost beyond recognition.

---

[9] The problem of whether the arrow moves in an instant or is at rest is of metaphysical importance since the answer is neutral in respect to classical mechanics, Michael J. White, *The continuous and the discrete: ancient physical theories from a contemporary perspective*, Oxford: Clarendon Press 1992, 177-179.

[10] Cf. G.E.L. Owen, Zeno and the mathematicians [1957], in W.C. Salmon (ed.), *Zeno's paradoxes*, Indianapolis: Hackett 2001, 157-163.

[11] Charles Dunan, *Les arguments de Zénon d'Élée contre le mouvement*, Paris: Alcan 1884, 8. A more charitable assessment states that it was an elementary blunder but a valuable one since it made Aristotle to analyze the problem of relative motion, N. Booth, Zeno's paradoxes, *Journal of Hellenic Studies* 77 (1957), 195.

[12] Lee, *op. cit.*, 89; Owen, *op. cit.*, 148-150.

[13] John Immerwahr, An interpretation of Zeno's stadium paradox, *Phronesis* 23 (1978), 22-26; Caveing, *op. cit.*, 114-116.

[14] Ferber, *op. cit.*, 25-26.

6.5 PLURALITY

In the four paradoxes presented so far, Zeno used infinity as the key element of proving that motion is impossible, i.e., that the Eleatic Being is motionless. Infinity is also used to prove that plurality is impossible, i.e., that the Eleatic Being is one and indivisible.

An approach to argue against plurality of things is given in a nutshell by Themistius who said that Zeno "tried to show that being is one, from its being continuous and indivisible, saying that if it were divided, it would not be truly one, because of infinite divisibility of bodies" (*In Phys.* 12.1-4). The crux of the argument is infinite divisibility, although from the summary given by Themistius it is not quite clear how it works against the existence of plurality. Would the impossibility of infinite divisibility prevent him from proving his contention? And why is infinite divisibility so detrimental to the existence of plurality?

In a more detailed description we read that the division into infinity can be performed by bisections which result either in indivisible elements, or in points with no magnitude. "Both [conclusions] are absurd." Why are they absurd? Indivisibles are impossible if infinite divisibility is seriously considered, since the body should be divisible everywhere, not only in one of its parts. Therefore, by the nature of infinite divisibility, it is absurd to assume the existence of indivisibles. In that case, infinite divisibility dissolves a body into dimensionless points, and the body becomes "composed of what is nothing, since if something [still] exists, the division process is not yet everywhere complete" (Simplicius, *In Phys.* 139.27-140.5).[15] That is, if a body can be divided into dimensionless points, then, apparently, the body itself is dimensionless because how can something composed of dimensionless points have dimension? Zeno touched here upon the problem of the composition of the continuum, which to this day is somewhat puzzling. A line interval is composed of points, and yet the interval has a nonzero length. How can that be? The reason is that besides the existence of an infinity of points their organization must also be considered, which, in set theory, is reflected in the distinction made between ordinal numbers and cardinal numbers. The problem is also investigated in measure theory. For example, an uncountable set of points in an interval [A, B] can have its combined length equal to zero, as, for instance, in the Cantor set;[16] but when irrational numbers are taken together in [A, B] in their natural order, they also form an uncountable set, but this time of a nonzero length. Therefore, it is only partially true that adding zeros gives a total equal to zero, since if a lot of zeros are added, we can get a nonzero value; however, just "a lot" is not sufficient; it also matters how these zeros (or, points) are organized. Therefore, only with the help of measure theory we can today refute Zeno's pronouncement that "what has no magnitude nor thickness nor mass," i.e., a point, when added to something, "would not increase its size; for a magnitude that is nothing [= zero] is incapable, when added, of resulting in an increase in magnitude" (B2, A21). One point added to anything certainly does not change the magnitude (the measure) of the result, but when points representing irrational numbers are added, each point being of zero magnitude – that it is quite another matter.

It is interesting that Zeno argued against plurality using the assumption of infinite divisibility and thus the zero magnitude of points. For him, it was not only that these points did not amount to a nonzero magnitude, but he even considered such points as nonexistent: what exists must have an extension, so, points are just illusory entities (A21).[17] In only one place we find an argument against plurality which does not rely on infinite divisibility. The argument says that if Socrates is an element of a plurality of people, i.e., if Socrates is treated atomically, we can say that he is pallid, snub-nosed, and interested in philosophy, which means that Socrates is really a plurality (Philoponus, *In Phys.* 42.24-27). The argument is pedestrian, very unconvincing, and different in character from other Zenonian arguments, therefore, it is quite plausible that it was not made by Zeno at all.[18] If so, then Zeno really did not address the problem of plurality in the atomistic universe. It may very well be that, in his absorption in the problem of infinity, he simply overlooked the case. Or, since the assumption of infinite divisibility is not his but his adversaries', he did not encounter a criticism of the Eleatic ontology from the perspective of the atomistic universe. Such a perspective was proposed in a mature form only later by the atomists.

6.6 INFINITY

A common feature of Zeno's arguments against motion and against plurality is the assumption of infinite divisibility. It would appear that infinity has no room in Eleatic philosophy. However, the arguments do not condemn infinity in all its manifestations, they only condemn infinity by division. Infinity by addition has a prominent position in Eleatic ontology. As shown in the previous chapter, Being is characterized by infinite extratemporal duration, or the infinite present. There is no end to its existence just as there is no beginning to it. Being is ungenerated and immutable, and its coming-to-being or perishing surely would mean its mutability. Also, as previously discussed, infinity was not altogether negative for the Pythagoreans, either. It was a desirable attribute of reality, a material to form the world, and a desirable attribute of individual human beings in the hereafter. Because of the insistence on the ontological role of numbers in the Pythagorean system and the first contrasting pair in the table of opposites, infinity acquired a negative air with the Pythagoreans. Probably, the discovery of the problem of incommensurability of a side of the square with its diagonal was instrumental to this negative view of infinity. Infinity by division, or rather its paradoxical character, was instrumental in providing arguments in favor of the validity of Eleatic ontology. Whether this paradoxical character led directly to founding this ontology is rather uncertain, but surely was used to fortify it. The Eleatic system was an answer to the counterintuitive consequences of infinity by division, which came at the price of the system being equally counterintuitive. Aristotle will try to harness infinity by division not by giving it an illusory character as the Eleatics did, but, so to speak, semi-illusory status by introducing the concept of potential infinity. Before that happened, infinity by division was elevated to new heights by Anaxagoras.

---

[15] Simplicius said that Porphyry had ascribed this reasoning to Parmenides, but Simplicius advocated following Alexander in ascribing it to Zeno (140.21-23).

[16] Divide an interval [A, B] into three even parts and remove the middle part without its endpoints. Divide each of the remaining two subintervals into three parts, and remove the middle thirds without their endpoints. Keep dividing each remaining subinterval into three parts and removing the open middle subinterval, into infinity. What is left, is the Cantor set, which is nowhere dense, uncountable, and has Lebesgue measure 0.

[17] Nothingness (e.g., of magnitude) and nonexistence were identified by the Eleatics, 28B6.2, 30B7.7, Hermann Fränkel, Zeno of Elea's attacks on plurality, *American Journal of Philology* 63 (1942), 21-22.

[18] Lee, *op. cit.*, 27-29.

Also, it is interesting to notice that these paradoxes had virtually no effect on the development of science, in particular, mathematics. Euclid, who within one generation or so from Aristotle wrote the *Elements*, had no compunction about using infinity. It was taken for granted and used freely without apparent intimidation coming from Zeno's *aporiai*. We can find in this an analogy to the impact of Gödel's theorems on the development of mathematics. These theorems changed the philosophy of mathematics, undermining Hilbert's formalism in mathematics, but mathematics itself did not cease to exist – quite the opposite. These theorems are more problematic to philosophers than to mathematicians. So it was with Zeno: he strongly affected Aristotle, but mathematics fared very well in spite of Zeno's paradoxes.

7 EMPEDOCLES

Parmenides was the first philosopher who formulated the principle that nothing can come from nothing (B8.7-10).[1] The principle also became the cornerstone of Empedocles' philosophy (B11, B12, B17.30-33), which had very important consequences for his understanding of infinity, or at least the temporal infinity of the world: the world is eternal.

7.1 ETERNAL CYCLES

The thesis that nothing comes from nothing can be interpreted in the Eleatic fashion by stating that there is no change in the world at all or by permitting change in unchangeable substrate. The latter way has been used since the time of the Milesians. And thus, for example, the *Apeiron* is the source of everything that exists and everything eventually returns to the *Apeiron*. That is, everything is a manifestation of the underlying *Apeiron*, everything is simply a form of one substrate. Empedocles basically followed the Milesian line of reasoning having explicitly stated the nothing comes from nothing principle and extended it to the nothing can be destroyed principle (B12). However, there are four material substrates in the Empedoclean universe: fire, water, earth, and air. All of them have been tried separately by his predecessors as single *archai* (and earth in conjunction with water by Xenophanes), and Empedocles may have found it somewhat unintuitive that, for instance, hard stones are but forms of tenuous air (Anaximenes) or of fleeting fire (Heraclitus) or even fluid water (Thales), or that fire is but a form of water (Heraclitus). Therefore, he might have simply collected all four recognized *archai* and given them independent existence as four substrates called the roots or elements. Now, everything that exists is a combination of the four elements created by two other *archai*, love and strife, the former being the attraction force and the latter being the separation force. Actually, both love and strife are forces of both attraction and separation, but they work differently. Love attracts elements and eventually creates a homogeneous mixture of elements of equal proportion, i.e., it separates elements from things if the proportion is not even. Strife separates elements to eventually create areas containing the same elements, i.e., it attracts the same elements to common areas.[2]

The nothing comes from nothing principle is only meaningful if the being itself (the Eleatics) or a substrate is eternal, that is, temporal (or, for Parmenides, the existential) infinity of the world is the ontological starting point. It is no different for Empedocles, as repeatedly confirmed by the sources. Empedocles declared that fire, water, earth, and air are eternal and everything else is made out of these four elements (30A5); these four elements are eternal, not made despite of numerical increase and decrease; they unite to form the one and separate from the one to form the many (31A28); as eternal, they are gods (A40). Love and strife are also immortal and ungenerated and never "will unutterable eternity be emptied of these two" (B16).

---

[1] Alexander P.D. Mourelatos, Pre-Socratic origins of the principle that there are no origins from nothing, *Journal of Philosophy* 78 (1981), 649.

[2] Cf. Joel Wilcox, "Whole-natured forms" in Empedocles's cosmic cycle, in A. Preus (ed.), *Essays in ancient Greek philosophy VI: before Plato*, Albany: State University of New York Press 2001, 117.

There are two general tendencies in the universal changes that take place interchangeably due to the eternal enmity of love and strife. In one phase, love becomes stronger and stronger, suppressing the influence of strife which, at the end of the cycle, results in all elements being perfectly mixed and forming the universal homogeneous sphere. Afterwards, the cosmic pendulum swings in favor of strife which becomes stronger and stronger, suppressing the influence of love which finds refuge at the limits of the universe. At the end of this phase, the universe still seems to be of spherical shape with spherical layers of unmixed elements.[3] After this, love gains on strength to subdue strife and form the homogeneous sphere, and the process continues from eternity to eternity through the cosmic carousel driven by the two Manichean forces. The world as we know it comes into being when love and strife are of comparable strength and things in the world can retain their cohesion due to the power of love and they can retain their individuality due to the power of strife. There is nothing loving in the total power of love, since everything disintegrates into a uniform mixture of elements. There is nothing desirable in the total power of strife, either, since the world disintegrates into elements kept separately. Both love and strife are needed to build the world, and both love and strife are, when too powerful, destructive to the world.

7.2 THE SPHERE

Temporal infinity of all things as a whole – their eternity – is almost taken for granted by Empedocles. There are eternal elements and eternal powers that form the foundation on which mutability of the world is based. There is apparently only one world – our world – which comes into being and perishes to be generated again, not necessarily in the same form as before. An infinity of worlds can thus be understood in strictly successive sense. However, spatial infinity is quite another matter. Unlike his predecessors, Empedocles denied the spatial infinity of the universe and became a forerunner of Aristotle in his hostility to the idea that the world can be infinitely extended in space. He directed a clear criticism of the idea of an infinite earth espoused by Xenophanes: "If the depth of earth and abundant aether are infinite as the tongues in the mouths of many have vainly poured

---

[3] "Four concentric circles divide presently the sphere," Jean Bollack, *Empédocle*, Paris: Les Iditions de Minuit 1965, v. 1, 169; Denis O'Brien, *Empedocles' cosmic cycle*, Cambridge: The University Press 1969, 146-155; Brad Inwood, *The poem of Empedocles*, Toronto: University of Toronto Press 2001, 52.

forth, seeing little of the whole" (B39). It appears that for Empedocles, it was just a vain speculation to propose the infinity of the earth since experience cannot confirm it: one can see only a little of the whole. However, the very same criticism can be directed back at Empedocles who hardly saw everything in the vastness of the universe during the eternal changes resulting from the conflict of love and strife – unless, of course, his claim that he was an immortal god (B112) and thus had enough time to investigate the universe is taken at face value.

Because the universe is of finite size, it appears that any infinitistic allusions should be interpreted in finitistic manner. When Empedocles spoke about the universal law of justice, "a law for all [which] extends through the broad air and through the ἄπλετος light" (B135), the word *apletos* should be understood as measureless or immeasurable, boundless or infinite.[4] "The *apletos* height of air" (B17.18) should be taken in similar fashion. However, there is a curious description of the sphere, the final product of love, the total mixture of all four elements. Empedocles said that it is "on all sides equal and altogether ἀπείρων, the rounded sphere, exultant in surrounding solitude" (B28). Side by side, the description says that the sphere is spherical, i.e., finite, well-delimited, and yet at the same time that it is *apeiron*. Does this mean that it is infinite? The sphere should be understood as literally spherical, limited in space, and therefore not infinite. This much is clear from his criticism of the infinity of earth, from the name itself: *sphairos*, and from the redundant emphasis that the sphere is rounded (B27, B28). Therefore, if infinity is meant here, it must not be infinite magnitude of the sphere. Why is it seen as infinite, then? One explanation states that the sphere "has no spatial starting or stopping point" and "there are no internal frontiers dividing one element off from another,"[5] which probably means that one could wander on the surface of the sphere never reaching the end and cruise inside the sphere never finding a borderline between constitutive elements. The elements are lost, as it were, in infinite division. In a similar spirit, it was stated that "the sphere limits itself on the outside. On the inside, the borderlines would indicate members/limbs."[6] One interesting way of explaining the phenomenon of the finite and, at the same time, the infinite character of the sphere is by pointing to its being the *plenum*: "there is nothing outside of it [the sphere] since it has embraced everything in itself, and thus one cannot speak about any limit which would separate it from anything external."[7] The sphere would be infinite since in its finitude it exhausts everything that is. But it seems that Empedocles wanted to set his sphere apart from Parmenides' Being by stressing the fact of the infinite divisibility of elements to contrast it with the indivisibility of Parmenides' Being (28B8.22).

---

[4] M.R. Wright, *Empedocles: the extant fragments*, New Haven: Yale University Press 1981, 285; Nicolaus van der Ben, *The proem of Empedocles' Peri physios*, Amsterdam: Grηner 1975, 197.

[5] Wright, *op. cit.*, 190; Leo Sweeney, *Infinity in the Presocratics*, The Hague: Martinus Nijhoff 1972, 143.

[6] Bollack, *op. cit.*, v. 3.1, 139; an allusion to B29: "From its back no twin branching arms are swinging; it has no feet or swift-moving knees, or sexual organ shaggy. It is a sphere, on all sides equal unto itself." Therefore, the *apeironic* character of the sphere means that it is without structure, v. 1, 88, 199.

[7] Vojtěch Hladký, Empedokleův Sfairos, *Filosoficky Časopis* 54 (2006), 399.

The sphere is not infinitely extended in space, but it is apparently characterized by another infinitistic aspect, namely the lack of smallest elements, atoms, that is, the elements are infinitely divisible. It should not be surprising that at the time when Zeno was using the paradox of infinite divisibility to defend Parmenides, Empedocles may have used infinite divisibility as a positive characteristic of the sphere. He mentioned the fact that elements are running through one another (B17.34, B21.13), i.e., he referred to the interpenetration of elements.[8] What is a mixture composing the sphere and how thoroughly it is mixed? We need to remember that blood is composed of all four elements in *almost* the same proportions (A98), whereas exactly equal proportions are to be found in the sphere (B17.27). Bones and flesh are also mixtures of four elements "in exact proportions" (A78). What would be the difference between the perfection of mixture in the sphere and in bones and flesh? Maybe "in exact proportions" should be weakened to approximate proportions,[9] but maybe the difference is in the thoroughness of mixture: bones and flesh have equal amounts of the four elements that are not mixed quite perfectly so that on the microscopic level, little pieces of each element could be distinguished. What about the sphere? It can be claimed that regardless of the microscopic level to which we descend, no single bits of any elements can be distinguished: the sphere is always a homogeneous mass of something which looks like one element, a perfect fusion of the four elements. Probably, the realization of this possibility led Philoponus to the statement that under the rule of love, the many becomes one and the sphere becomes perfect when the appearance of particular elements cannot be distinguished whereby the sphere becomes qualityless (*In De gen. et corr.* 19.3-9 = A41). This may be a paradox of infinite divisibility of the elements. When a perfect interpenetration of elements is created due to their infinite divisibility and thorough work of love, the separate properties of the elements cancel one another, at least in their manifestation, so that the mixture appears to be devoid of qualities. It must, however, be assumed against Philoponus that even in this perfect mixture, somehow the nature of the four elements is not undermined so that they can come back to life under the control of strife.[10]

If the mixture in the sphere is infinitely divisible, then, in the manner of Zeno's argument, it is possible to show that an infinity of different objects can be created out of this mass in spite of the fact that the volume of the mass is finite. Thus, the finite sphere can be pronounced to be infinite.

---

[8] Bollock, *op. cit.*, v. 3.1, 79.

[9] As suggested by Harald A.T. Reiche, *Empedocles' mixture, Eudoxan astronomy and Aristotle's connate pneuma*, Amsterdam: Hakkert 1960, 54 note 1.

[10] Therefore, a claim is made that this is not absolute homogeneity, because this mixture contains the possibility of an emergence of heterogeneous elements, Johann Ch. Lüth, *Die Struktur des Wirklichen im empedokleischen System "Über die Natur"*, Meisenheim: Hain 1970, 31. Aristotle also suggested that divisibility stops at a certain point (*De caelo* 305a1-4 = A43a), and this seems to be accepted as Empedocles' own view by Wright, *op. cit.*, 37-38.

8 ANAXAGORAS

The idea of infinity so strongly present in the systems before Anaxagoras, reaches its apex in Anaxagoras' philosophy. It is fully present in his concept of Mind: Mind is infinite in the sense that it is not limited by space since it exists everywhere where matter is (59B14) which is also infinite and composed of an infinite number of parts (B1, B2); Mind is also infinite in respect to duration by being eternal, not limited by time (B14); Mind's power is infinite since matter which it controls is infinite (A15, A57, B12). However, Anaxagoras' physics, with its double infinity – the infinity that has no upper limit and no lower limit – is unmatched in ancient thought. But arguably the most puzzling part of the system is his claim that everything has a portion of everything else, which can be termed the everything in everything principle (EEP).[1]

8.1 PHYSICS

Anaxagoras' system is based on a few assumptions that are more or less clearly formulated in the extant fragments and in the doxographic tradition.[2]

The first assumption states that the motion in nature is not an appearance, as the Eleatics claimed. Senses are in this respect reliable, and we should not reject their testimony.

The second assumption also relies on the senses and is about the infinity of mutually irreducible substances. We perceive a variety of phenomena in nature, and this variety is generalized into their inexhaustibility. The reality is an infinite multiplicity, not unity, πάντα, not πᾶν as the Eleatics thought. The four basic elements introduced by Empedocles are insufficient to explain this multiplicity. It is not enough to say that water, air, earth, and fire exist; we have to assume that there is a separate substance for bones, flesh, cartilage, hair, gold, stone, etc.

The third assumption says that what exists was never created out of nothing and will never be annihilated (B17). According to Aristotle, the commonly held assumption that nothing comes from nothing allows Anaxagoras to assume the existence of an infinity of principles, that is, homoiomeries (*Phys.* 187a26-30 = A52). The primordial mixture that existed before the creation of the world already contained everything, if only in the form of seeds (B1, B4). All the elements were mixed together, and from this mixture the cosmos emerged through the action of Mind that introduced an order into the mixture (B12). But how can this be reconciled with the existence of change in nature without accepting Parmenides' solution that, in reality, there is no change? The third assumption can be considered as the principle that prohibits any qualitative changes – substances cannot turn into one another, although quantitative changes are possible, including changes in space and time. In reality, nothing new comes into being because everything already exists in the world. This is expressed in the central and, at the same time, most controversial thesis that is implied by the three assumptions, namely that in everything there is a share of everything, which is stated six times by Anaxagoras (B6, B11, B12). No substance exists in pure form since each particle of substance, regardless of its size, contains a share of everything else. The thesis "everything in everything" is associated with the thesis "like from like" (A45.17), that is, with the thesis about nonexistence of qualitative changes, which is ascribed to Anaxagoras because of the question he asked: "how can hair come into being from non-hair?" (B10). "Everything in everything" can be understood as the presence of each substance in any particle of any other substance and so "everything is everywhere,"[3] and regardless of which part of the infinite cosmos is scrutinized, all substances can be found there.

---

[1] The principle was also briefly stated by Diogenes of Sinope (DL 6.73).

[2] Cf. Sven-Tage Teodorsson, *Anaxagoras' theory of matter*, Göteborg: Acta Universitatis Gothoburgensis 1982, 23-24; George B. Kerferd, Anaxagoras and the concept of matter before Aristotle [1969], in A.P.D. Mourelatos (ed.), *The Pre-Socratics*, Garden City: Anchor Books 1974, 490-491.

The fourth assumption, connected with the third, is the principle of infinite divisibility of all things: "there is no smallest among the small" (B3); "there can be no smallest" (B6). The principle is closely connected with the views of Zeno of Elea although it is uncertain whether Zeno wrote before or after Anaxagoras. Anaxagoras did not have any problem with infinite divisibility, which sometimes is taken as an indication that he wrote before Zeno or at least did not know of Zeno's paradoxes and that these paradoxes are Zeno's reaction to the views of Anaxagoras. However, it is possible that for Anaxagoras, Zeno's arguments were simply unconvincing and not worth mentioning. Anaxagoras may not have had any problem in adopting the technique of infinite divisibility that Zeno used, but he could have considered the philosophical application of the technique as groundless.

In spite of the divisibility principle, attempts to interpret Anaxagoras' theory of matter frequently gravitate in the direction of a molecular interpretation by trying to force it into the framework of a theory that accepts an existence of the smallest indivisible elements. This tendency exists at least since Aristotle,[4] who did his best to remove any traces of infinity in nature, and sometimes this theory is accepted by today's interpreters of Anaxagoras.

The fifth assumption is the principle of the difference of proportion: a character of a particular thing depends of what predominates in it (B12). Existing things obviously differ from one another, but what is the nature of this difference if everything is in everything? The answer is: there is a difference in proportion of the constitutive elements.

---

[3] Teodorsson, *op. cit.*, 75.

[4] In *Physics* 1.5, Aristotle purposely omits the infinite divisibility of matter, which allows him to ridicule Anaxagoras' theory. However, the molecular interpretation introduced by Aristotle, and repeated for centuries because of Aristotle's authority, is not mentioned by Plato and Xenophon.

In the beginning, there were two beings: omnipotent and omniscient Mind and the primordial mixture (σύμμιξις). Everything was together in the mixture. To the senses, no substance was distinguishable from another substance, and so an impression could arise that the primordial mixture was a uniform substance. However, omnipotent Mind initiated a motion of the mixture whereby the rare was separated from the dense, and the light from the heavy, and thus objects filling the world were created. Mind has a full control over all changes in the world, over what was, is, and will be. Mind is not mixed with anything and exercises this control from the outside. However, Mind cannot accomplish full separation of substances since the EEP never ceases to be valid.

As mentioned, the primordial mixture contains everything which exists as complete or yet to be completed. The contents of the mixture can be divided into three groups: the opposites, the four Empedoclean elements, and seeds.

The first group includes the wet and the dry, the hot and the cold, and the bright and the dark. Today, these elements are called the opposites, after Aristotle, which does not prejudge whether they are material substances or attributes. Anaxagoras treated substances on a par with opposites (B4), and so the cold and the hot, etc. seem to be to him substances endowed with these attributes.[5]

The next group is the basic elements listed by Empedocles. However, Anaxagoras mentions only three of them, namely air, fire[6] and earth, as included in the primordial mixture (B1, B2, B4, B15).[7] These elements are as primordial and as compound as other natural substances. There is no difference in status between fire and air on the one hand and bone and flesh on the other.

Finally, the third group is seeds (σπέρματα). There is a problem with interpreting them because Anaxagoras mentions them only twice and only in one fragment without explaining what seeds exactly are. Sometimes an opinion is voiced that one important philosophical innovation of Anaxagoras is the introduction of this concept. The opinion is argued by Anaxagoras' remark about seeds of all things, which is considered to be an obvious generalization of the observation about the nature of seeds in the world of plants and animals.

There is, however, some tension between the normal understanding of seeds and the EEP. According to B4, the primordial mixture contains an infinity of seeds and also in everything, there are seeds of all things. Because each seed leads to the generation of a particular thing, the thing has to contain seeds of all other things, and thus each seed contains seeds of all things; each of these seeds, in turn, contains seeds of all things and so on ad infinitum, which is in agreement with the EEP. Seeds can be thus considered as particles, but not indivisible particles, not the simplest elements, but compound entities. "*Sperma*, in Greek, is as much a biological as a botanical term: where the word 'seed' suggests particles to us, the word *sperma* would not have done so to a Greek … To say that X contains seeds of Y need mean no more than that Y can grow from X."[8] But this does not contradict the fact that X is infinitely divisible and contains everything else.

---

[5] W.K.C. Guthrie, *A history of Greek philosophy*, Cambridge: Cambridge University Press 1965, v. 2, 285-286; Teodorsson, *op. cit.*, 78.

[6] In reality, Anaxagoras talks about aether, not fire, but according to Aristotle, Anaxagoras identified aether with fire (*De caelo* 270b24-25 = A73).

[7] Although water is not listed as an element of the primordial mixture, it is mentioned in B16 as a stage of the process of transformation of elements into other elements, and so Anaxagoras saw in water one of the natural substances, cf. Teodorsson, *op. cit.*, 43 note 107, 77 note 164.

It is important to mention the position of homoiomeries in Anaxagoras' system. Anaxagoras himself does not use the term. The term ὁμοιομερῆ was probably coined by Aristotle, who applies it to substances divisible into parts composed of the same substance. A catalog of homoiomeries in *Meteorology* 388a13-20 includes both metals (gold, copper, tin, etc.), and tissues (skin, hair, bones, bark, wood, leaves, etc.). If we accept the term homoiomeries, then the term does not refer to substances composed only of substances of the same kind[9] since this would contradict the EEP. Homoiomeric substances are the ones that are similar in different parts. And so granite is a homoiomeric substance although it is composed principally of quartz, feldspar, and mica, and if we concentrate on a particle of mica in granite, we would reach another homoiomeric substance. Similarly, dough with raisins is a substance and so is ore and chocolate fudge. Therefore, the following alternative has to be rejected: because of interpretative problems with homoiomeries, "we have to be *in favor of* our [Anaxagoras'] texts and *against* homoiomeries or *in favor of* homoiomeries and modify our texts."[10] The alternative disappears when we do not impose on the term homoiomeries the meaning that is alien to Anaxagoras' system while retaining the term.

The primordial mixture should be understood as a mixture of all substances in the same proportion in each particle of the mixture. Anaxagoras says that the primordial mixture contained the greatest quantity of aether and air (B1). This prevalence should be understood in this way that if the mixture were divided into sufficiently small parts, say, into 1 mm^3 cubes (in the primordial mixture "nothing was manifest" (B1)), then in each of these parts there would be more aether and air than other substances with respect to volume, weight, or something else (they prevailed "both in number and in size" (B1)). If this was the case with aether and air, it is likely that among other substances there were uneven proportions of similar kind.

Because the mixture also contains seeds of all things and seeds are characterized by some kind of order (the tendency to generate larger entities), the prime mixture is not tantamount to the existence of total chaos, to the state of perfect entropy, but it is characterized by a uniform distribution of all substances, where the uniformity includes a prevalence of some substances over others, always in the same proportions. It is thus a thoroughly mixed mass of the existing substances, opposites, and seeds.

In this mixture, Mind introduces an order. Because of the EEP, as in the pri-

---

[8] Jonathan Barnes, *The Presocratic philosophers*, London: Routledge 1982, 323.

[9] Homoiomereity does not mean that "however far one divides any of the elements, the parts will always be like the whole," Guthrie, *op. cit.*, 282-283; similarly Kerferd, *op. cit.*, 495.

[10] Jean Zafiropulo, *Anaxagore de Clazomène*, Paris: Les Belles Lettres 1948, 278-279. A contradiction between the existence of homoiomeries and the EEP is also seen by Francis M. Cornford, Anaxagoras' theory of matter, *Classical Quarterly* 24 (1930), 14.

mordial mixture, so in the created world, substances never exist in pure form, none of them can be completely separated from other substances. If some objects in the world are seen as different than others, this is because of the principle of different proportion. Therefore, bone and stone are seen as bone and stone because in a given part of space, there is greater concentration of particles of bone and stone although they also contain traces of all other substances. This seems to have also been the understanding behind the distinction between elemental and common substances (E- and C-substances). The former cannot be separated because "there is a portion (μοῖρα) of every E-substance in every piece of matter, but not in isolable portion; for however far you subdivide any piece of matter each subdivision will contain the same number of E-ingredients as the original. One must never speak of *pieces* or *bits* of E-substances, but only of *portions* (or proportions or percentages) of E-substances in pieces or bits of C-substances."[11] E-substances cannot be separated from other substances, just as the left side cannot be separated from the right side and the north pole of a magnet from its south pole. Therefore, the criticism of E-substances, namely that in Anaxagoras' system there is no room for unmixed substances, is ill-directed since the elemental character of E-substances does not mean that they are not mixed: E-substances are mixtures which for explanatory purposes are treated separately although, in reality, they do not exist in separation, just as in Aristotle's system (or in any nominalist system) properties do not exist in separation from objects.[12]

It is stated that proportions of a given substance are independent of the size of a portion of substance; that is, for example, for bones, its proportion exceeds the proportion of other substances whereby bone is perceived as bone. "This proportion would be maintained however small a speck of bone we imagine, even far below the level of perception, for 'the large and the small have proportions equal in number' (B6)."[13] However, we need not forget that we are referring to infinite amounts, which does not mean that proportions were retained. Although in each particle of bone there are infinitely many particles of bone, this particle includes also an infinite number of particles of any other of the infinitely many substances. Therefore, on the perceptual level, bone has greater proportion of the bony substance than other substances in respect to weight or volume although on some level particles of bone can have a greater proportion of other substances, and so if they could be perceived, they would not be perceived as bone.

Is it possible that bone can cease to be bone on some level? To answer it, let us use the example of a particle of water. The particle is composed of two particles (atoms) of flammable gas and a particle of gas that maintains fire, nevertheless, as a result of binding of two atoms of hydrogen and one atom of oxygen, we obtain liquid which is used to extinguish fire. Anaxagoras could have used a similar example. We eat lettuce, carrots, and cake, and these substances create in the body bones, sinew, cartilage, etc. And so, on some level, bone appears as carrot and lettuce, but on the perceptual level, bone is seen as bone because on this level bony substance has a greater proportion than other substances. If we used a sufficiently strong microscope, we could see particles of lettuce and carrot in bone. If the process of division of particles of bone is continued, we obtain something that is not bone but, e.g., lettuce. This does not mean that bone was reduced to particles of lettuce since if the process is continued, on a certain level of division of particles of lettuce contained in bone, we obtain bone. And so, not each particle of bone is bone. Also, the reduction of bone to something else is possible on some level of division, but on some other level of division we obtain bone again, which, in turn, can be reduced to something else when the process is continued. Therefore, with some reservations, we could agree with the statement that seeds are something elementary.[14] They are elementary in the sense that they cause an entity to be what it is. However, they are not elementary in the sense of being indivisible: seeds, just as everything else, are infinitely divisible and contain everything in them.

---

[11] Colin Strang, The physical theory of Anaxagoras, *Archiv für Geschichte der Philosophie* 45 (1963), 102.

[12] Cf. Teodorsson, *op. cit.*, 58; Patricia Curd, *Anaxagoras of Clazomenae*, Toronto: University of Toronto Press 2007, 190-191.

[13] Guthrie, *op. cit.*, 297. Similarly, Gregory Vlastos, The physical theory of Anaxagoras [1950], in Mourelatos, *op. cit.*, 475-476.

The statements just made are a consequence of the EEP and the principle of infinite divisibility. Anaxagoras' theory is exceptional in that respect, and we have to wait until Leibniz whose vision of infinite divisibility of matter matches Anaxagoras'. As Leibniz says, "there is no particle of matter which does not contain a world of innumerable creatures" and "each portion of matter may be conceived of as a garden full of plants and as a pond full of fish. But each branch of the plant, each member of the animal, each drop of its humors is also such a garden or such a pond" and the air between plants and the water between fish contain more plants and fish, "but for the most part so tiny as to be imperceptible to us."[15] Today, that vision can to a large extent be illustrated with the concept of fractals.

8.2 THE EVERYTHING IN EVERYTHING PRINCIPLE

The EEP has always intrigued researchers who advocated different approaches to reconcile it with other parts of Anaxagoras' physics.

In spite of some attempts to dismiss the EEP as superfluous,[16] the EEP is a

---

[14] Teodorsson, *op. cit.*, 31.

[15] Letter to Arnauld, 9.X.1687, in G.W. Leibniz, *Philosophical papers and letters*, Dordrecht: Reidel 1969, 345; *Monadology* 66-67; Adam Drozdek, Leibniz: struggles with infinity, *Studies in Logic, Grammar, and Rhetoric* 14 (1997), 60-61.

[16] Even "grotesquely superfluous," Cornford, *op. cit.*, 18; the bipartite paper of Cornford is replete with invectives: the EEP is absurd and gratuitous, p. 16, 20, 92, grotesque and ridiculous, p. 91; Anaxagoras would be muddleheaded not to see his own principles Cornford's way, p. 14. For a critique of Cornford's interpretation, see Vlastos, *op. cit.*, 482-483; R. Mathewson, Aristotle and Anaxagoras: an examination of Cornford's interpretation, *Classical Quarterly* 8 (1958), 67-81.

fundamental principle of Anaxagoras' physics, an "untouchable principle,"[17] and cannot be discarded to simplify our interpretation of Anaxagoras' physics. However, its meaning can be variously modified.

One explanation of the EEP is by limiting the scope of "everything" referred to by the principle. It is said, for example, that there is no flesh, blood, or bone in bread and other foods, and it is not true that "the ones consisted of the others, but that both consisted of the same thirds – i.e., of these ultimate elements: rare- and dense-*moiras*" and other *moiras*.[18] Substances differ by different proportions of the *moiras*. It can now be claimed that everything is in everything and, in particular, "any seed contains any other simply through the fact that it contains their necessary ingredients in portions which, however small, would still suffice to produce any of these seeds in correspondingly minute form."[19] But in this way, Anaxagoras' physics hardly becomes distinguished. The same can already be found in the Milesians when they introduce one substrate for all things. Water was chosen by Thales because water is "easily formed into each different thing": into slime and earth when compacted, into air when exhaled, and "the finest part is kindled from air into aether" (Heraclitus Homericus, *Quest. Hom.* 22); therefore, slime is in air and vice versa because they can be converted into one another through the mediation of water. Anaximander proposes the mechanism of separation and destruction in physics to explain the emergence and destruction of physical entities. Therefore, each physical entity is potentially in any other entity because they can be transformed into one another through the *Apeiron* as an intermediary. It is similarly with Anaximenes' *aer* and the condensation-rarefaction mechanism he uses, and with Heraclitus' fire. Also, the four elements of Empedocles constitute everything that exists in the world having different proportions of the elements, and so anything can be transformed into anything using the material constituting a particular object.

In this interpretation, trivially, everything is in everything because the material forming any object can be restructured and remolded to create something else. In this way, the presence of everything in everything is merely *in potentia* and thus hardly noteworthy. In this way, the important structural aspect of the EEP is dissolved. No seed really contains any other seed in a complete, structured form. There is only a *possibility* of transforming a seed into another seed, which, presumably, can only be done by Mind because only Mind "with numerical exactness knows the combining-ratios of the elements for the *mereias* of every stuff."[20]

Instead of excluding seeds from the scope of application of the EEP, an attempt can be made to apply the EEP to seeds only. In particular, it is said that homoiomeries are "simple and indivisible material elements and each of these elements is homogenous" and the EEP applies only to compound objects.[21] However, even assuming that the homoiomeries are truly simple and indivisible, it is far from clear in what sense one compound object contains another compound object.

---

[17] As aptly phrased by René Lefebvre, Matière et résolution: Anaxagore et ses interprètes, *Revue Philosophique* (1996), 51.

[18] Felix M. Cleve, *The philosophy of Anaxagoras*, The Hague: Nijhoff 1973, 87.

[19] Vlastos, *op. cit.*, 484. See also Malcolm Schofield, *An essay on Anaxagoras*, Cambridge: Cambridge University Press 1980, 107-121; John Burnet, *Early Greek philosophy*, Cleveland: Meridian Books 1957 [1920], 263 (everything contains a portion of opposites only). Cf. the claim that the EEP does not apply to seeds, Eric Lewis, Anaxagoras and the seeds of a physical theory, *Apeiron* 33 (2000), 18-19

[20] Cleve, *op. cit.*, 88.

A next interpretation of the EEP to consider is the blending model, according to which different substances are so thoroughly mixed that different portions have the same proportion and consistency as the whole. A cup of water mixed with coffee extract is as homogeneous as the contents of this cup poured into two smaller cups, to use Kerferd's example.[22] It should be obvious that the validity of the EEP breaks down at the level of coffee and water particles: these particles reside side by side, and so a particle of one substance is devoid of an admixture of any other substance. The problem can be solved in two ways. One way is to permit interpenetration.[23] If it is possible that two substances can occupy the same space, then the EEP is vindicated. However, it is unlikely that Anaxagoras would accept the possibility of interpenetration of substances. The possibility is put forth much later by the Stoics and, arguably, on theological rather than physical grounds: God should be present in all of the universe, which was possible when God, a material being, could occupy the same place as other material entities. Moreover, occupying the same space by two objects is not quite the same as one object being in another object. Also, interpenetration allows for the possibility of many bodies residing in the same place, but is not intimately connected with the fact that everything has a portion of everything. Also, two seeds may be of different shape and size, they may overlap by occupying the same space, but there are portions of one seed not included in another seed, neither seed being another seed, not to mention everything being in everything. Finally, many different objects can reside in the same space, but not every portion of space has to have all substances and seeds and so, interpenetration only allows for the EEP to be valid, but does not require it.

Another way of rescuing the blending model is to assume that the particles are sizeless. It is stated that "Anaxagoras may have thought in terms of an infinitesimal powder," that is, it is as if substances were ground into powder of infinitely small particles.[24] This is carrying the mixture model to the extreme. However, in this interpretation, there are smallest parts for which there is nothing smaller, which is in violation of the expressly stated condition in B3. Moreover, by Sorabji's own admission, "arrangement would create a problem: the sizeless units of bone, gold and honey would need to be arranged in relation to each other

---

[21] Charles M. Zevort, *Anaxagore*, Paris: Joubert 1843, 62, 71. It seems that the interpretation of the EEP along similar lines can be detected in the statement that "in all objects are to be found all seed kinds," David Sider, *The fragments of Anaxagoras*, Meisenheim/G.: Hain 1981, 134.

[22] Kerferd, *op. cit.*, 499.

[23] Lewis, *op. cit.*, 12-13.

[24] Richard Sorabji, *Matter, space, and motion*, Ithaca: Cornell University Press 1988, 64. The possibility to extend the validity of Anaxagoras' physical principles "down to the infinitesimal" is also mentioned by Kerferd, *op. cit.*, 499.

with all the necessary ratios."[25] This points to the problem from which the entire blending model suffers: the model does not account for the structure of seeds which exist already in the primordial mixture and which also include everything and are included in everything. How can that be explained if the world is seen as a mixture of liquids or powders, as a mixture of smaller and smaller particles that are eventually devoid of size? The blending model stresses the unstructured character of the world. It remains powerless in grappling with its orderliness, hierarchical character, and omnipresence of structure.

It seems that the proper interpretation of the EEP should consider seriously the recursive character of Anaxagoras' theory of matter. He states that "there is no smallest among the small, but always a smaller; ... moreover, there is always [something] larger than the large" (B3). Everything is infinitely divisible, everything is infinitely magnifiable. Infinite divisibility can be understood as simple division of an entity into sections with no regard of the parts composing the entity; such a division would be applicable to everything, in particular, to homoiomeries. However, division can also be understood mereologically, as division of compound objects into parts, these parts being decomposable into their parts, etc., without reaching atomic parts. Such a recursive aspect of Anaxagoras' system was often recognized,[26] but it was frequently glossed over and not used in the interpretation of the EEP.[27]

Inherently recursive objects are described by the modern theory of fractals which are self-replicating structures that in each of their parts, however small, repeat the form of the whole. The interpretation to follow is inspired by this theory.

As remarked at the beginning, one assumption of Anaxagoras' system is the principle of infinite divisibility of matter. It is possible that Anaxagoras used the reasoning that can be found in Zeno's paradoxes. As discussed in chapter 6, the first paradox, the dichotomy, states that an interval AB can be divided into a half, the second half can be divided into a half, the second half of the second half can be divided into a half, etc., so that we obtain an infinite sequence of divisions. It seems that Anaxagoras generalized the method of division used in the dichotomy, which allowed him to accept the EEP. The interval AB is divided into an infinite sequence of intervals, among which the first, AC, is a half of AB, the second, CD is the fourth of AB, the third – the eighth of AB, etc. (Fig. 1).

---

[25] Sorabji, *op. cit.*, 65.

[26] Vlastos, *op. cit.*, 480-481; Mathewson, *op. cit.*, 75; Kerferd, *op. cit.*, 501-502.

[27] The only exception is an interpretation proposed by Gareth B. Matthews, On the idea of there being something of everything in everything, *Analysis* 62 (2002), 1-4; but, Matthews does not address the problem of seeds being in seeds focusing on substances being in substances (he uses an example of milk being decomposable into buttermilk and butter). However, his solution explicitly uses recursion, which would make possible to fit the structural aspect of matter into his solution.

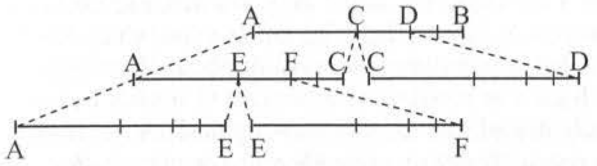


Figure 1

Thus, the interval AB includes an infinity of different intervals, each of different length than any other. This division can be continued for each of these subintervals. For example, the subinterval AC included in AB can be divided into an infinity of subintervals, among which the first, AE, is a half of AC, the second, EF, is a fourth of AC, etc. The same can be done with the second subinterval included in AB, the interval CD (which is shown in Fig. 1, where interval AC and CD are magnified). The interval AC is thus composed of an infinite number of subintervals. And because the process of division of AC was the same as applied to AB, then the first is an image of the second, that is, the subinterval AC of AB includes similar elements as interval AB, where the similarity consists in the same ratio of length of a particular subinterval to the length of the entire interval. For instance, the interval AE in AC is a counterpart of AC in AB because AE is a half of AC, and so is AC in AB. In this sense, the division of each interval can be continued in infinity for each interval and at each level of division similar elements can be found. In this way, we obtain an infinite tree of divisions, in which at each level there is an infinity of subintervals included in the intervals from the previous level. This renders an infinite hierarchy of divisions that repeat divisions from previous levels. Therefore, everything is in everything, everything has a share of everything.

This reasoning can be generalized to a plane, where a division of, for example, a square is a function of division of both sides at the same time. In this way, we obtain a division of the square into an infinity of rectangles, each of which can be divided into an infinity of rectangles, each of which can be divided into an infinite number of rectangles and so on ad infinitum (Fig. 2). Thereby we guarantee that the rectangles on the diagonal are of the same shape as the rectangle that is being divided, and among other rectangles there are such that are similar to the rectangles previously divided. Everything is in everything.

It is clear that the same procedure can be applied in the three dimensional space by dividing, for example, a cube into an infinity of rectangular prisms, each of which can be divided into an infinity of rectangular prisms etc. so that we would find along the way that the shapes of resulting rectangular prisms are similar to the shapes of previously divided rectangular prisms (Fig. 3). Everything is in everything.

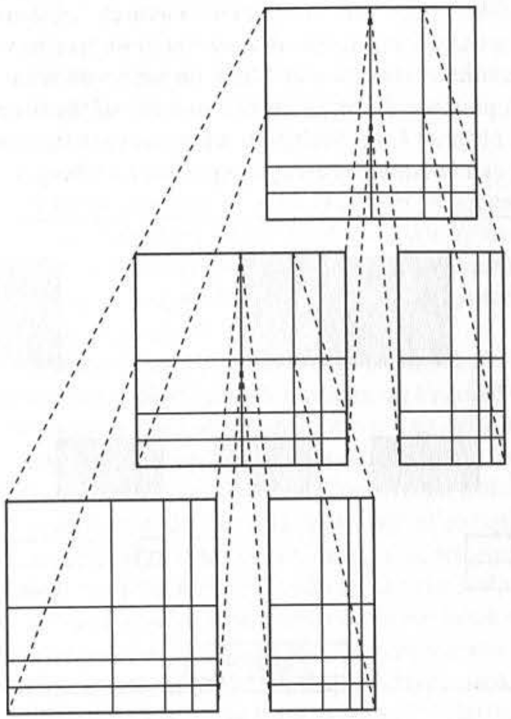


Figure 2

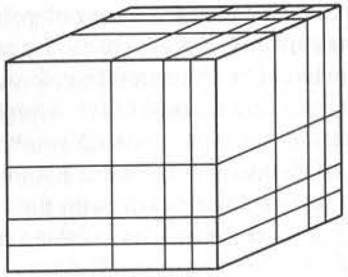


Figure 3

The construction can be somewhat simplified by dividing a particular object (interval, rectangle, or rectangular prism) into a finite number of objects each of which can be divided into a finite number of objects, etc. The principle of division can be changed from one object to another: one object can be divided into two objects, another into three objects, yet another into four objects, etc., each of which is also divisible. In this way, we obtain a hierarchy of divisions. By starting with any object, e.g., a rectangle, we obtain an infinite tree in which the number of rectangles on each level is finite and which on each path from the root includes a rectangle of each possible shape an infinite number of times and each seed also an infinite number of times (Fig. 4a depicts a part close to the root of such a tree). Only in this way it can be said that everything is in everything.

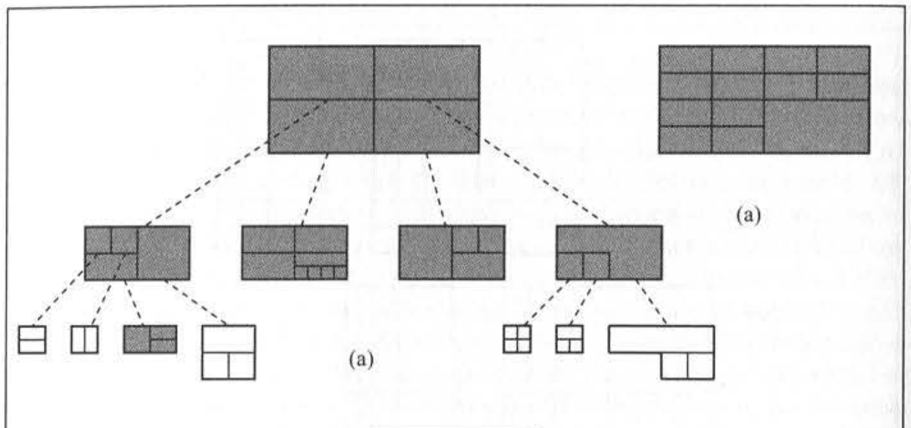


Figure 4

The construction just presented can be used to illustrate a possible way to interpret seeds. According to Anaxagoras, there are seeds of all things, there are seeds of such natural substances as bone, gold, stone, etc. Moreover, seeds "have all possible shapes (ἰδέαι), colors (χροιαί) and tastes (ἡδοναί)" (B4). The question is how, for example, gold is generated by a seed of gold? Unfortunately, Anaxagoras does not provide any details. According to the normal meaning of the word seed, something bigger should be generated from it. According to the assumptions of Anaxagoras' system, there are no qualitative changes in the world. Therefore, a seed of a natural substance can be considered an object which among first level parts has a prevalence of elements of the same nature as the object itself. Let, for instance, an element of gold be a rectangle with the ratio 2:1. An entity is a seed of gold if among its first level parts, most of them are rectangles with the ratio 2:1, and the number of such parts can be different in different seeds. The prevalence in a certain entity can be expressed in terms of the quantitative predominance of elements (and so, in the second rectangle from the left in second row in Fig. 4a, the ratio of elements of gold to elements of non-gold is 3:8, that is, the rectangle could not be a seed of gold) or in terms of the greater area occupied by 2:1 rectangles to the area occupied by other rectangles (for the rectangle just indicated, the ratio is 3:1, that is, the entire rectangle is a seed of gold). The first level parts do not have to be of the same size: Fig. 4a shows at the top a seed of gold composed of four elements of gold, Fig. 4b illustrates a seed of gold that contains

thirteen such elements. Elements of these elements can be different and on a certain level of division they must be different for the EEP to hold. By nature, each 2:1 rectangle has golden color, although the color can disappear – and eventually must disappear – when dividing a golden element into smaller elements. We can now view the growth of the seed of gold into portions of gold in such a way that elements of gold gravitate toward a seed of gold.

It is noteworthy that this interpretation can be significantly simplified by considering each 2:1 rectangle as a seed of gold. However, it appears that in this way it would lose the specific character of the seed as an entity that gives growth to something which contains non-seeds as well. Also, by contrasting earth and seeds (B4), Anaxagoras seems to suggest that earth is not composed only of seeds of earth, an idea which can be extended to other substances.

The situation becomes slightly more complicated in the case of seeds of compound entities such as seeds of plants, animals, and people. In this case, more than in the case of seeds of natural substances, we can rely on the assumption, which very likely was accepted by Anaxagoras, namely the theory of preformation. According to the then popular theory of Hippocrates, which relies on the like from like principle, seeds contain essential elements of an organism that grows from the seed.[28] In a scholion to Gregory of Nazianzus, we read that according to Anaxagoras, "all things have been mixed with one another, but they become separated as they grow, for in the same seed (γονή) there are hairs and nails and veins and arteries and sinews and bones; it is so that they are imperceptible because the small size of their parts, but when growing, they gradually separate. 'Because – as he says – how can hair come into being from non-hair?'" (B10). The scholion lists some tissues present in organisms. It seems, however, that organization of the tissues should also be added: an unorganized collection of fragments of bones, hairs, etc. does not constitute a seed, and the tissues also have to be suitably organized. It thus seems that, at least, a seed is a grown organism in miniature approximately, or a configuration of elements that inevitably lead to the generation of an organism. If, for example, such a seed is of the form shown in Fig. 5a and is composed of two kinds of tissue (one represented by a 4:1 rectangle and another by two squares), the role of a seed is fulfilled when it develops into a grown organism. This takes place by assimilating tissues from the environment and from food in such a way that the growing organism approximately retains the original form. Because growth takes place by transferring matter to seeds, growth is possible by decomposition of other individuals. Two stages of such a growth are shown in Fig. 5bc. It should be noted that the shape of an organism can sometimes undergo significant changes (as in the case of the transformation of a caterpillar into a butterfly), and the process of growth may consist not only of assimilation of new particles of tissues, but also of replacing existing particles with other particles (when, e.g., a tree loses leaves, which grow back next spring).

---

[28] *The seed*, *The nature of the child*, and *Diseases* iv 32-34; *Airs, waters, places* 14.

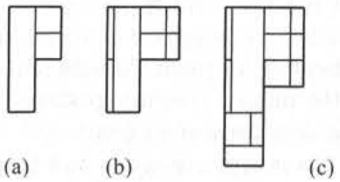


Figure 5

8.3 THE INFINITELY LARGE

Anaxagoras' physics recognizes not only infinity by division, but also infinity by addition. At the beginning, matter as a mixture of all things was infinite. This is stated at the beginning of Anaxagoras' *Physics*, in B1, where the word *apeiron* appears three times, thereby pointing to Anaximander.[29] At a certain point, Mind started the ordering process by bringing about locally rotation that has become wider and wider, like circles on water (B4, B13). Because matter is infinite in quantity, this process of widening the scope of the rotation will never be ended. More and more matter will be processed first by the rotation and then by separation and thus organized and ordered by the supervising Mind, but there will always be an infinite leftover. Creation is thus an ongoing process during which the mixture is transformed not just into single world, but into multiple worlds (B4a), prospectively, into infinitely many words in the process which will never end.

Anaxagoras constantly connects infinity by addition with infinity by division. He says that "of the large, there is always the larger, and it is equal to the small in quantity" (B3). It seems to mean that there are as many levels of the infinitely large as of the infinitely small, that is, infinitely many levels in both directions. When dividing matter into smaller portions, no ultimate limit will be encountered. When dividing matter, the starting point is a portion of matter which was obtained from some larger portion with no upper limit ever being reached since matter is infinite in quantity.

Some statements made by Anaxagoras bear strong resemblance to the ones made by Zeno to the extent that some dependence between them is universally recognized. However, it is not clear who influence whom: whether Anaxagoras' system spurred Zeno's response[30] or whether Anaxagoras' theory was a response

---

[29] As rhetorically asked by Gigon, "whence could the idea of the undifferentiated and infinite mass as [constituting] the *arche* stem if not from Anaximander's *apeiron*?" Olog Gigon, Zu Anaxagoras, *Philologus* 91 (1936), 9.

[30] E.g., David J. Furley, *Two studies in the Greek atomists*, Princeton: Princeton University Press 1967, 76; Brad Inwood, Anaxagoras and infinite divisibility, *Illinois Classical Studies* 11 (1986), 20.

to Zeno's paradoxes.[31] However, the important think is the common theme, namely, the infinite. Infinity was taken for granted by Anaximander and made a foundational concept of his theory. Only later problems emerged which were best expressed by Zeno in his four *aporiai* and the paradoxes of plurality. Infinity is present in the Eleatic system, but it is tamed by limiting it only to infinity by addition so that the majestically static and indivisible Being does not admit of infinity by division. Anaxagoras, on the other hand, embraced the concept of infinity both by addition and by division creating an impressive system in which Anaximander's infinity is included and, in addition to it, projected into the infinitely small.

---

[31] E.g., Gigon, *op. cit.*, 4-5, 15-16; Theo G. Sinnige, *Matter and infinity in the Presocratic schools and* Plato, Assen: van Gorcum 1971, 129-137.

9 THE ATOMISTS

The view of the atomists is encapsulated in the statement of Democritus that things are "by convention sweet and by convention bitter, by convention hot, by convention cold, by convention color, but, in truth, [there are only] atoms and void" (68B9, B125, A1.44-45). The way of opinion relies on the testimony of senses alone and takes the sensory data as the expression of the essence of reality. But the senses provide unreliable or even, as picturesquely phrased, bastardly information (B11). However, the way of truth should see with the eye of the mind that behind the multitude of manifestations there are only atoms and the void. The Eleatic vision is projected from the macro level to the micro level: for the Eleatics there was one true immutable Being, to the atomists, there are imperceptibly small immutable (although movable) atoms and the immutable void. The macro level was, as it were, emptied and only the void was retained on that level. On the other hand, the micro level was populated with atoms beyond counting.

9.1 ATOMS AND THE VOID

In the universe of the atomists, there is space, infinite in extent, called the void, nothing, and the *apeiron* (A37). The void is considered to be nonbeing and privation (Themistius, *In Phys.* 129.8-9). Also, there are infinitely many indivisible atoms and they have all, thus infinitely many, shapes (*schemata*) and sizes. All existing things are formed by separation, joining together, and alteration by order and arrangement of atoms (67A9). Atoms are called things, solids, being. They are so small that they are imperceptible. They travel through space and collide, thereby forming different bodies by remaining dovetailed and interlocked since some atoms are uneven, some with hooks, some concave, some convex, etc. with innumerable differences (68A37).

Atoms are changeless because of their solidity (A1.44). Atoms are made of uniform material and when a piece of this material forms a particular atom, the atom is eternal. Why does a particular atom have a particular form and shape? No explanation is provided and the eternity of the world can be used to explain that away. The world was always filled with the same atoms; they are what they are because they are eternal; there was no first cause that brought them into being or formed them in a particular way from preexisting matter, therefore, because of their eternity, an attempt to explain their shapes and sizes can be considered as not altogether cogent. An explanation that atoms have infinite shapes since, there is no more reason for them to have one shape than another (67A8, 68A38), which is also the explanation given for their infinity (68A38), is very unhelpful. An argument could be made that positing the infinity of atoms, and the infinity of their shapes and sizes, really shows an inability to provide a more intelligible explanation of the existence of our world. The atomists wanted to explain the world around them and the diverse phenomena in it. They proposed atoms as building blocks, but could not decide what shapes would account for the phenomena best, so they decided that all shapes have their embodiment in some atoms (A37) and thus had to have the infinity of atoms. It may be just a sign of theoretical desperation: if the number of shapes and sizes of atoms cannot be harnessed in explanatory assumptions, all of them are allowed. It appears that for the atomists it was, as it were, an embarrassment of riches since they did try to be more specific in their explanations about the shapes of atoms. This is best testified to by a long testimony of Theophrastus and explanations which match particular shapes with particular sensory perceptions (A135). That is, a finite number of shapes could be sufficient for explanatory purposes and this is actually the conclusion later reached by Epicurus. Moreover, Democritus implicitly defied the assumption of permitting all atom shapes in his cone problems which cannot be reproduced when this assumption is made.

There is an infinity of sizes of atoms (A1.44) because atoms come in all sizes (A37). This would mean that, when taken literally, the number of sizes of atoms, and thus the number of atoms is nondenumerable and that the number of atoms cannot be accommodated in three-dimensional Euclidean space.[1] Therefore, all sizes of atoms cannot be permitted, but the denumerable number of possible atom sizes can easily be allowed.

Atoms are said to be imperceptible (67A7, 68A37), that is, their sizes do not exceed the level of human perceptibility. There is no account why that should be, presumably, there would be a law built into the universe which would set the limit on the size of atoms. However, this seems to be contradicted by testimonies that, according to Democritus, atoms can be extremely large (68A43) and be even as big as a world (A47). This, however, does not have to mean that atoms are visible. If there are infinite sizes, then some atoms can be gigantic in comparison with other atoms, even if both atoms cannot be seen by the human eye. But maybe they can be seen by a nonhuman eye, an eye of a being in another world? According to the atomists, there is an infinity of worlds and it is possible that the perception of beings in other worlds is different from ours and fit to perceive even atoms. The atomists stressed the fact that different universes can be of a very different nature. In fact, "there are infinitely many worlds differing in size" (A40), and the remark that an atom can be as big as a world can simply be turned around to mean that one world can be as small as an atom from another world.[2] This view may be a bow to Anaxagoras' theory which allows for entities to be of any size, regardless of how small the size is – including the size of the entire world. That would mean that in the Democritean universe there may be worlds of the size of an atom, i.e., worlds that cannot be seen by the human eye, a view not at all outlandish when we consider a very populous world of bacteria that eludes the perceptible range of human unaided senses. Also, if atoms were visible, then their random motion would very likely be disruptive to the environment. Worlds are said to be destroyed by clashing with one another (A40.3) and certainly the world composed of large atoms would destroy a world made out of small ones.[3]

---

[1] Assume that some coordinate system with three axes has been established. Associate with each atom a triple <x, y, z> of rational numbers signifying coordinates of a point inside the atom. Because the number of rational numbers, and thus, the number of such triples is countable, the number of all atoms, if infinite, is countable.

[2] See also Charles Mugler, Les théories de la vie et de la conscience chez Démocrite, *Revue de Philologie* 33 (1959), 11-15; Salomo Luria, Zwei Demokrit-Studien, in J. Mau, E.G. Schmidt, *Isonomia*, Berlin: Akademie-Verlag 1964, 42 note 2; Peter J. Bicknell, Kosmos-sized atoms in Democritus, *Apeiron* 15 (1981), 138-139; C[hristopher] C.W. Taylor, *The atomists: Leucippus and Democritus*, Toronto: University of Toronto Press 1999, 173-174; Elizabeth Asmis, *Epicurus' scientific method*, Ithaca: Cornell University Press 1984, 273.

An infinity of atoms move in continual motion in the infinite space from eternity to eternity (67A8, A16, 68A56) in the eternally existing void since, as assumed by the atomists, it is obvious that time has no beginning (68A71). The atoms "travel in the void overtaking one another and colliding and some rebounded at random, but others became entangled with one another through the fitting together of shapes, sizes, arrangements and orders, and thus they brought compounds together" (67A14). Everything that exists is the result of particular arrangements of atoms that cluster together, forming objects, groups of objects and entire worlds.

The first problem that the infinity of space poses is the question of motion. Why should atoms move rather than stand still suspended in the infinite immensity of space? Aristotle later explained movement by natural motion which could only be meaningful in a finite universe: the natural motion for light bodies is to move upwards, whereas heavy bodies naturally move downwards. The explanation was not without its problems because it was endangered with circularity: why do bodies move downward? Because they are heavy? Which bodies are heavy? The ones that move down. Democritus could not use directions as guides for motion of atoms, which Aristotle found absurd – how can one think that there is no up or down in the cosmos (*De caelo* 308a17-19)? Very likely because of his criticism, this was rectified – or rather artificially incorporated in his system – by Epicurus. Democritus' space was isotropic, infinitely and uniformly extended in each direction which no natural guide in this borderless immensity where the atoms should go. Therefore, it appears that weight is an irrelevant attribute of atoms when explaining motion in the void at the time when they are not part of any world.[4] Atoms have only shape and size, according to Democritus; later, Epicurus also distinguished weight as atoms' property to explain motion (A47).

How did Democritus explained motion? For him, the motion of atoms was the result of mutual impact in infinite space, the result of the force termed a blow (A47). Aristotle criticized him for not explaining the origin of their motion (67A15, A18). In fact, Democritus did not provide any explanation. Aristotle, although his universe was eternal, felt compelled to use the unmoved mover and the desire for this mover as the origin of motion. Democritus before him used the temporal infinity of the universe to his advantage: the atoms move because they always moved and, apparently, establishing the origin of motion would counter the eternity of the universe. Therefore, eternity of the world saved Democritus from pursuing the problem of the origin of motion and, in fact, such pursuit could even be considered meaningless (cf. A65). He just accepted the motion of atoms as a fact. They move because they move and because they have always moved.

Is being in motion a natural property of atoms? This is where the problem of motion in an infinitely existing universe can become harder than in a created universe. Democritus is said to believe that atoms are by nature immobile but are moved by a blow (A47). If so, what is the origin of the energy required for the blows? It has to come from the outside of atoms and the recourse to eternity of the universe is of little help since the statement that atoms have always moved by blows and the energy for the blows is not the atoms' means that there are three principles in the universe: the void, atoms, and the energy required for motion. Only a little better would be an explanation that atoms are by nature mobile and this energy is their attribute.[5] Would it be possible to believe that there is enough energy in each atom so that it can be in motion from eternity to eternity? Not much energy would be needed to move minuscule atoms, but if this need is extended to eternity, then the question arises: is this possible without any renewal of energy? Maybe in the Democritean world the energy could be transferred from one atom to another during impact without any loss, unless a recourse to infinity is made yet again: there is an infinity of atoms and thus the total amount of energy for all atoms combined could also be infinite;[6] therefore, a loss of finite amounts of energy would not affect the infinite total. Actually, even an infinite loss of energy does not have to be detrimental to the infinite total.

9.2 INFINITY OF WORLDS

Both Leucippus and Democritus believed that there are an infinite number of worlds (A1.44, A43, A81, 12A17). This is the result of the isotropic character of space. Since, as rhetorically asked by Philoponus, for what reason a world would exist in one part of the void and not in the other? Therefore, if there is a world in

---

[3] Charles Mugler, Sur quelques particularités de l'atomisme ancien, *Revue de Philology* 27 (1953), 146-147; as soberly remarked by Mugler, when we start seeing atoms, that would not be for long.

[4] Cf. Denis O'Brien, *Democritus, weight and size: an exercise in the reconstruction of early Greek philosophy*, Paris: Les Belles Lettres 1981, 331-332.

---

[5] Such a possibility may be suggested by the fact that for the atomists the soul is the source of motion of the body (67A28), the reason being that the soul is made out of small round atoms and the sphere is the most mobile of shapes (A101); i.e., the round atoms seem to have the inner source of motion that is imparted to the soul. A possibility for atoms having an inner source of motion is mentioned by Rudolf Löbl, *Demokrits Atomphysik*, Darmstadt: Wissenschaftliche Buchgesellschaft 1987, 108, 115-116, and Taylor, *op. cit.*, 194; René Kayser, *Die Urbewegung der Atome bei Leukipp und Demokrit: eine Studie über die Zusammenhänge von Ontologie und Physik im abderitischen Materialismus*, Luxembourg: Societé Luxembourgeoise de Philosophie 1997, 160.

[6] It could be finite even for infinity of atoms as a version of Zeno' bisection argument can show.

one part of the void, there are other worlds also in its other parts and because of the infinity of the void, the infinite number of world follows (*In Phys.* 405.23-27).

The worlds are all different and in different stages of development: some are with a sun, some without, and the suns can be of different sizes, with different configurations of planetary orbits, if there are any planets. Some are with animals, some are animal-free. Some are of a very sophisticated structure, some are simple, unelaborated collections of simple objects. Some worlds have just been generated, some are at the peak of their development, and some are about to perish. They are unevenly spaced in the void. The worlds destroy one another by collision (A40), presumably, breaking one another into the atoms that constituted them and the atoms later become building blocks of other worlds. Therefore, all worlds "become the same as to kind (εἶδος), not as to number" (A82).

The cosmogonic process is rather complicated. The process begins when a large number of atoms "become separated from the infinity [of atoms] … into a great void" to form a swirl in which, through collisions and circling, like atoms are assembling with like atoms. A spherical structure emerges with small atoms at the periphery and larger atoms congregating and interlocking inside (67A1.31). From the sphere, a membrane is separated.[7] The membrane becomes thinner by the atoms' outflow toward the center, whereby the earth comes into being. The membrane makes up for its loss of atoms by attracting atoms on the outside through its rotating motion (67A1.32).

It is rather puzzling how these stages could be explained merely by the fact that atoms randomly impact one another in the void. At the very inception, a large mass of atoms has to be separated from other atoms randomly crisscrossing the void. How can such a separation take place randomly? Afterwards, the separated mass forms a rotating swirl. How is this possible that the random motion of all the atoms is suddenly organized into rotation of the mass? In the swirl, at first, like atoms are separated with like atoms. How can that happen? The atoms should hold due to interlocking, i.e., atoms with hooks can interlock, but this can hardly be the case for other shapes. Why should round atoms be congregating together and cube atoms together? It seems that much more stable structures could emerge when unlike atoms are put with unlike atoms, e.g., atoms with concavities into which spherical atoms could fit. Then, small atoms are squeezed out to the outside of the emerging sphere and the remaining atoms go downward. Why should that happen? Is it because of their weight? However, weight is supposedly an immaterial attribute of atoms. There are even testimonies that atoms should be considered weightless (A47). If so, whence the sudden appearance of weight? It seems that weight is always the property of atoms, but it does not matter if atoms are analyzed in the context of the entire isotropic void. However, inside a high concentration of atoms, weight has, well, some weight. Weight matters in a dense mass of atoms and determines their movement accordingly. But the outer membrane? There is no conceivable reason for it to exist. To explain its existence, an orderly – spherical – congregation of interlocking atoms is required. The atoms, however, are of different weights since the membrane constantly loses atoms and acquires new ones. How can it maintain its consistency? Atomic motion can hardly explain that.

This indicates that the atomists have to go beyond the mere assumption of the existence of the void and randomly moving atoms in it. Many times the atomists mention necessity and it appears that the like to like principle, the law of membrane, the law of rotation of the swirl, and possibly some other laws come into the province of necessity. "Just like the coming into being of the cosmos, so does its growth, decay, and destruction take place according to some necessity, the nature of which [Leucippus] does not explain" (67A1.33, A10). The law of necessity is thus just as important as the other two *archai*, the void and atoms, the law which encompasses all the laws that allow for all the transformations in all the worlds with the possible exception of the random motion of atoms. Unless the necessity is to be considered as another name for randomness, the latter designation to be used as an expression of our ignorance of the nature of orderliness of the universe determined by necessity (Theodoret, *Graec. aff. cur.* 6.15) or just a shorthand: Democritus spoke about random events, but in his more detailed discussion he was more specific about causes which were far from random (68A68). It is of little help when Democritus equated necessity with the cosmic swirl (A1.45) since it would leave out the laws leading to the emergence of the swirl outside necessity.[8] It is also of little help when he tried to explain necessity as meaning impact and motion and a blow of matter (A66), but hardly only random motion could be meant here if necessity is to account for the force of attraction, repulsion, and the force responsible for particular organization of atoms which is at work during the formation of worlds. All the transformations are thus anything but random and we call them random since we cannot determine the laws that stand behind them. After all, Leucippus in his only extant statement said that "nothing takes place in vain, but everything for a reason and by necessity" (67B2).

9.3 INFINITE DIVISIBILITY

The presence of infinity in the atomists' system is not limited to the infinitely large (infinite space, infinite number of atoms and their shapes, infinite worlds, infinite existence of the universe). If also includes the infinitely small.

Atoms by definition are indivisible entities because of their solidity. They seem to have been introduced as an ontological principle exactly to prevent the possibility of infinite divisibility in order to avoid counterintuitive consequences of Zeno's paradoxes. They constitute the limit to which matter can be divided. However, there is no smallest magnitude, according to Democritus (A48a), i.e., each magnitude can be divided infinitely, at least in theory. The size of an atom can be so divided, but not the atom itself. It is the nature of atoms to resist any attempt of breaking them into pieces. It has been said that the cause of the indivisibility of atoms (primary bodies) is their small size and partlessness, whereas Epicurus denied partlessness and said that atoms are uncuttable since they cannot be affected (67A13). However, atoms can be large, even of the size of a world; therefore, their size cannot be a good enough reason for their stability. Neither is their apparent partlessness since they do have parts: atoms with hooks have a body and a hook; a cubical atom has six sides which, not being identical with one another, constitute parts of the atom, even if only conceptual parts, even if no force in the universe can make them exist in separation. Already early commentators recognized the fact that atoms are indivisible in the sense that they have parts and size, but they cannot be divided because of their solidity and fullness (Simplicius, *In Phys.* 82.1-3). Atoms cannot be cut because they cannot be affected and contain no void (Eusebius, *Praep. ev.* 14.14.5).[9]

---

[7] Cf. textual problem concerning separation of membrane vs. separation of the sphere in Jula Kerschensteiner, Zu Leukippos A1, *Hermes* 87 (1959), 442-443.

[8] Cf. Taylor, *op. cit.*, 191.

There is here at least a terminological problem. For the atomists, what is truly continuous is indivisible (Themistius, *In Phys.* 123.18-20) and atoms are the only continuous objects (Simplicius, *In De caelo* 609.19-20). Is any magnitude divisible? Apparently it is, if it is not continuous. And what would such magnitude be? Maybe a fragment of space. However, space, the void, is termed nonbeing, therefore, magnitudes would also be nonbeings. But the void is hardly a nonbeing in the literal sense; the void is a being whose nature is fundamentally different from the being, or rather, the beings, i.e., atoms. That much, in fact was stated by Leucippus: "being [= atoms] exists no more than nonbeing [= the void] and both alike are causes of things that come into existence" (67A8). Therefore, magnitudes in some sense do exist, after all atoms are separated by the void and the separation distances could, at least in theory, be measured. But they are not continuous, at least not in the same sense as atoms are. Apparently, for the atomists only solidity can be continuous and the void cannot. Would that mean that some distance *d* between two atoms cannot be divided into two halves? Hardly. Therefore, it appears that simply another term should be applied to parts of the void and to theoretical objects (mathematical lines, planes, etc.), not continuity, but something else which reflects their divisibility, maybe constancy or uninterruptedness.[10]

The problem with continuity does not end here. How is it that atoms form objects? Because they cluster together, interlock, mesh one with another so that these three differing characteristics of atoms are distinguished: rhythm, turning, and contact, i.e., shape, position, and arrangement (68A38, 67A6). But if atoms cluster together, they may touch one another and touching would mean that at least at the touching point two atoms become continuous and thus become one atom. This is hardly an option since for an eternally existing world all atoms

---

[9] Bodnár analyzed the argument of smallness and compactness (having no void) as explanations of atoms' indivisibility and found both of them wanting, István M. Bodnár, Atomic independence and indivisibility, in A. Preus (ed.), *Essays in ancient Greek philosophy VI: before Plato*, Albany: State University of New York Press 2001, 123-147.

[10] As pointed by Luria, Соломон Я. Лурье, *Демокрит: тексты, перевод, исследования*, Ленинград: Наука 1970, 467 comment on fr. 237, Democritus apparently distinguished two types of continuity: continuity by contact (τῇ ἁφῇ συνεχές, Aristotle, *Phys.* 203a22 = 59A45) and true continuity (κατὰ ἀλήθειαν συνεχές, Simplicius, *In De caelo*, 609.22). Uninterruptedness of space would belong to the second category.

would eventually create one immense atom. Therefore, there must always be a distance between atoms forming an object that prevents them from being continuous.[11] This is a very curious consequence of the atomists' concept of continuity. Atoms never touch one another and even during collision there exists a force due to which this is always a collision at a distance, small distance as it may be. Interlocked atoms are also separated by the void; hooks do not really touch what they are hooked to.[12] The problem is carried over to macro objects since they also cannot touch one another because that would mean physical contact of the constitutive atoms. There arises, therefore, a very counterintuitive picture of physical phenomena, all of them taking place at a distance: when an apple hits the ground, it never touches the ground, hovering above it at an imperceptibly small height; when chewing something, the teeth never come into contact with the food, because of a small distance between the teeth and the food. The senses turn everything into illusion: we think we touch something, but we really do not; we feel pain by thinking we got hit, but we were not hit; we sense the sour taste of some food, but the food never really touches our taste buds; we think we see something but the *eidola* emitted by perceptible bodies (67A29) never get in touch with our eyes, flying right through them without touching any atom in them; we think that a body floats on water since it is born by hot atoms (A62), but these hot atoms really have to act on the floating body at a distance, thereby making it float. Our body is untouchable to other bodies and their actions and yet the world largely seems to operate in unison with our actions, performed as though we truly knew what is afoot around us. How this happens, however, remains unexplainable by the atomists' accounts based on their principles. More, much more, has to be assumed to make such explanation intelligible and generously used infinity in the atomists' system is sometimes helpful in providing an explanation, but it also can be a hurdle. The explanation that this is all just an illusion, or rather convention, that there are no colors or tastes but just a motion of swirling atoms (A111), that these perceptions are empty states of the senses (SE 8.184) and there is no truth in the senses (B165), is not entirely satisfactory since there is some regularity and structure in that illusion which has to be accounted for, and the atomistic reductionism is unable to provide a suitable account.

By carrying to the extreme Parmenides' principle, that "it is the same to think/know (νοεῖν) and to be" (28B3),[13] the atomists introduced infinity of atoms moving in infinite space, infinity of atom shapes and sizes, and infinity of worlds since all of it could be thought, but although infinity acquired in their system a new prominence, it came at a cost of leaving unresolved finer issues. Infinity was used, as it were, as a sufficient key to ontology, but it was used too generously, even too recklessly, which created an immense universe with infinity of worlds, but additional principles are needed to explain the emergence of these worlds since infinity by itself, or rather infinity of randomly moving atoms in infinite space are far from sufficient to provide a satisfactory explanation of physical phenomena. This infinity has to be tamed, and such attempts were made by Plato, Aristotle, Epicurus, and the Stoics.

---

[11] Simplicius, *In Phys.* 648.11-13; Philoponus, *In De gen. et corr.* 160.7-11; Philoponus, *In Phys.* 494.19-25; the atomists also stated that no two atoms can be fused into one, 68A37, A42.

[12] Cf. Taylor, *op. cit.*, 187-188; S[alomo] Luria, Die Infinitesimalmethode der antiken Atomisten, *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik, Abt. B: Studien* 2 (1933), 155.

[13] Charles Mugler, L'isonomie des atomistes, *Revue de Philologie* 30 (1956), 236; "Leucippus postulated on the basis of the Eleatic principle of the correspondence between thought and reality, real existence, materialized in atoms, of this infinity of geometric forms that the spatial intuition reveals to us." Democritus extended it to the postulate of infinity of sizes, p. 243.

10 PLATO

Not infrequently the view on infinity in Greek philosophy in general and in Plato's system in particular is expressed in a statement that says that both for Plato and Aristotle "the finite represents a veritable, permanent being that is identical to itself, that is, intelligent, infinity is associated with becoming, change, that is, with the indeterminate and the unintelligible."[1] This view, however, is an Aristotelian legacy which is not entirely true even for Aristotle's system, and certainly not true for systems of other ancient thinkers. As for Plato, his system is founded on infinity. Infinity is a very foundation of his system, not as a separate concept, but as its indispensable basis and as a perspective that is inherent in this system. This is manifested in his ontology, epistemology, theology, and eschatology. Infinity is often taken for granted, something to base the system on, not as a problem to be solved, not as a concept to be constructed.

10.1 FINITE WORLD

According to the grand cosmogonic vision of the *Timaeus*, the reality before the creation of the world already had a complex structure and the process of creation was not creation out of nothing: the universe was formed by the eternally existing Demiurge from eternally existing prime matter in eternally existing space using eternal paradigms as models. The prime matter was a disorderly mass of unspecified structure out of which atoms of fire, air, water, and earth were formed as regular solids, and then the universe was shaped out of the masses of these atoms.

There is only one universe (*Tim.* 31a, 55cd) and this universe is shaped as a sphere (33b, 58a) and therefore is finite, but is the prime matter finite? Plato said that the Demiurge made the four constituents "as far as was possible, proportional to one another, so that what fire is to air, air is to water" (32b, 56c, 69b). If the amount of fire, air, and water were unlimited, then it would be meaningless to talk about proportionality. Only finite amounts can be constitutive of proportions, that is, ratios.[2] Moreover, the Demiurge built the finite universe "from all the fire, water and air and earth … leaving no part or power of any one of them out" (32c), which means that the amounts of these constituents had to be finite before the building started. These constituents, however, were made out of prime matter. Was all the prime matter used up in the process of constructing fire, water, air, and earth? Plato was unclear on this point. If the primal mass were infinite, then some of it would remain unused after the creation of the world. This could hardly be considered turning disorder of becoming into an order because the universe would be just a finite speck in comparison with an infinite ocean of the remaining chaos. Moreover, the infinity of the prime matter would mean that necessity, the power ruling over the prime matter, has to be infinite to cause the motion of this infinite mass. As such, necessity could easily overturn the work of the Demiurge and turn its orderliness quickly into disorder. Necessity would have to be separated from the world not to do it, but this would require infinite power of the Demiurge to make it possible. Therefore, the Demiurge would have to be infinite just for this one thing, namely to keep necessity at bay and away from the universe. But as a whole, reality would be highly unordered if within the primal chaos there would be only a small island of order: our universe. This seems like an unworthy result of a creative action of the "most perfect God" (68e). So it seems very natural for this God that the primal mass ruled by necessity first give order to "all these things" by turning them into atoms "and then out of them put together this universe" (69bc). This means that in Plato's universe, the Demiurge used all the prime matter when forming atoms of the fire, water, air, and earth, and then all these atoms were used in forming the world. Because the world is finite, the primal mass must have been finite as well. Moreover, because of the finite size of the primal mass, the domain of becoming does not require an infinite housing and hence space is also finite.[3] And one more argument: it is explicitly stated that there is nothing outside the world created by the Demiurge (33c), hence there is no prime matter left.

---

[1] P. Kucharski, L'idée d'infini en Grèce, in *L'infini et le réel*, Paris: Albin Michel 1955, 13-14.

[2] Even in the Cantorian set theory, proportions between transfinite cardinal numbers can be considered, at best, metaphorically.

Although the primal mass was finite, Plato seems to have suggested that the amount of triangles used to shape the prime matter into atoms is infinite. To explain a variety of objects in the world, Democritus assumed the existence of infinity of shapes among infinitely many atoms. Plato assumed the existence of only two different shapes of the elementary triangles out of which only four different types of atoms are built. How could he explain the presence of so many different varieties even among the same kind of bodies using only four different shapes of atoms? One explanation may use different proportions of atoms in different mixtures. But how can one explain different kinds of fire or earth (58c-59c)? To explain that, Plato assumed that originally elementary triangles are "both smaller and larger, numerically as many as there are varieties in the kinds." That is why when mixed together "they display an infinite diversity" (57d).[4] The problem with that solution is that whereas there is enough room for an infinite number of atoms in the infinite void of the Democritean universe, the finite Platonic universe apparently cannot accommodate an infinity of elementary triangles. How can we reconcile an actually infinite number of elementary triangles with the finite size of the universe?

The solution can be found in the proposal of Anaxagoras for whom the universe allows for infinite divisibility. Plato expressed in his dialogues his admiration for Anaxagoras (e.g., *Phaedo* 97cd, *Phaedrus* 270a), and it is quite possible that he implicitly agreed with his idea of infinite divisibility of matter.[5] For Plato, triangles were not divisible, but their sizes were. In this way, elementary triangles, although they do not exceed a certain size (none of them is visible individually, 56bc), can still be infinite in number in the finite universe if their sizes are generated using the bisection argument.

We may also add that the infinity of elementary triangles does not contradict the fact that the Demiurge distributed the four elemental masses proportionally in space. This would be impossible if proportionality were measured in terms of the numbers of elementary triangles. This proportionality is possible if the mass of triangles is meant (triangles appear to have a mass because the atoms have a mass, 56b) or their magnitude. This is in analogy to the comparability of the sizes of two line intervals, notwithstanding the fact that these intervals include an infinity of points.

Creation of the universe amounts to imposing an order on the finite prime matter that contains "certain traces" of fire, water, earth, and air. "The god put them together to be as perfect and excellent as possible, when they were not so before" (53b). This was done by molding atoms of these four elements using triangular shapes. As already mentioned, Plato assumed the existence of "infinite variety" of such triangles. But the infinity of triangles does not seem to be a critical factor for the creation and then for the existence of the universe. If there were a finite number of triangles, the universe would function just as well. The infinity of triangles, however, implies that the cognitive power of the Demiurge has to be infinite, because all the triangles have to be used in generation of the four masses of atoms.

If we regard the infinity of triangles to be gratuitous and imagine that the primal mass has only a finite number of triangles, then what can we say about the power of the Demiurge? May this power be finite? No, not in Plato's system. In this system, introducing a perfect order in the universe requires an infinite cause even though the dimension of the universe is limited. The Demiurge has to possess an infinite cognitive capacity to choose from an infinity of possibilities the one which is best. The Demiurge made the world by bringing chaos into order, that is, by fashioning with "forms and numbers" what was "without proportion and measure" (53ab), which brings us to the consideration of how many "forms and numbers" there are.

---

[3] Duhem's statement (Pierre Duhem, *Le système du monde*, Paris: Hermann 1954, v. 1, 37, 38) that space is "necessarily unlimited" is too generous, unless it is assumed that space spreads infinitely outside the spherical universe, which would violate Platonic statement on the non-existence of the void. Duhem tried to reconcile these two facts by saying that the void cannot exist only "in the bosom of the world" but in the rest of space "nothing exists, whereby it is void." Such a solution would require the outermost sphere of the universe to be impenetrable to the movements of what is in the world so that this sphere does not burst and the moving elements would not spill into the space outside this sphere. This solution is quite unlikely.

[4] At one point Plato mentioned that the isosceles right-angled triangle is of one nature (*physis*), whereas the scalene triangles are of infinitely many natures (54a), which may be taken to mean that there are infinitely many sizes of the latter triangles.

[5] An influence of Anaxagoras is also recognized by Charles Mugler, *Platon et la recherche mathématique de son époque*, Strasbourg 1948 [reprint, Naarden: Bekhoven 1969], 177; see also p. 119. An influence of Zeno does not seem very likely, since his *aporiai* had a completely different bent.

The remark on "forms and numbers" refers to all forms, i.e., ideas, to the realm of being, in general, and to geometrical shapes and numbers in particular, that is, to mathematical entities. How many forms are there and how specific are they? There is certainly an idea of triangle, but is there a separate idea of a scalene triangle, a right-angle triangle, a triangle with angles of 1, 1, and 178 degrees, a triangle with angles of 1, 2, and 177 degrees, etc., and for three particular angles, a triangle with combined length of all three sides equal to any number? In this way, there would be an infinity of ideas, and most of them would have no instantiation in the universe. This does not have to mean that universals, or ideas, without instances "would constitute a cosmic anomaly."[6] There may be ideas that have currently no instantiation, but they had it before (the idea of a dinosaur) or will have one in the future (a dwelling on the Moon). Because the world is presumably without end with respect to duration, each of the ideas may have a chance to be instantiated, at least once. There may be ideas that stand no chance of instantiation, at least in the material world, for example, mythological creatures. Is a thought about a Pegasus or a griffin such an instantiation? If so, then no cosmic anomaly would arise.

10.2 NUMBERS

Are numbers ideas? There were many attempts to reconstruct Plato's concept of the number. But because Plato wrote very little about the number, most of the reconstructions are based on Aristotle's *Metaphysics*.[7] Aristotle suggested a tripartite division of numbers into physical numbers, mathematical numbers, and number-ideas and not infrequently, this reconstruction is accepted as adequately reflecting Plato's understanding of numbers.[8] Mathematical numbers are to be compound entities composed of units and there can exist many copies of the same mathematical number; on the other hand, number-ideas are to be simple and unique. It seems, however, that Aristotle's division is forced into Plato's system. On the one hand, Plato mentioned myriads of units used to compute sums of numbers in philosophers' arithmetic as opposed to unequal units of practical arithmetic (*Philebus* 56de); he also mentioned numbers in which the unit is "each one equal to every other" (*Rep.* 526a). These statements apparently describe mathematical numbers. On the other hand, Plato wrote about the fact that "that which is to be one must share in oneness" and things that are to be two must share in twoness (*Phaedo* 101c). This statement is understood as referring to number-ideas. However, the problem with an apparent composition of mathematical numbers is not limited to numbers. There exists a square itself and a diagonal itself (*Rep.* 510d), that is, an idea of a square, a square-idea, and an idea of a diagonal. But both squares and diagonals are lines and as such, they are composed of points. Are, then, the ideal square and the ideal diagonal compound entities? Ideas are simple and if they were compound, they would cease to be ideas. But diagonality of a diagonal is not characterized by its being a continuum of points, but by its position in a figure, by its relation to other lines. Squareness of a square is not defined in terms of points, but in terms of angles and proportions of sides. The idea of a square abstracts from the fact that squares are points and characterizes squares in terms of what distinguishes them from other beings.[9] Similarly with all other beings. In Plato's universe, all entities in the world are made out of the four kinds of atoms, but the idea of a tree does not include the property of a tree being made out of these elements. And so it is with numbers. Twoness, the idea of two, is simple, not a collection of two units; it is a property of being composed of two parts, but it does not mean that the two itself includes two parts.

Moreover, Plato talked about an ideal unit, but the fact of the existence of infinity of numbers does not necessitate an existence of infinitely many identical units. We may assume an existence in Plato's world of ideas an idea of a root. Also, an idea of a tree and an idea of a bush would have to be defined in terms of, among other things, a root. Does this mean that there is a need for two ideas of a root so that the tree idea and the bush idea can be defined uniquely? Admittedly, there is in Plato's system not only a vexing problem of the nature of participation – how is it that the entities of the universe participate in their corresponding ideas – but also the problem of relations between ideas. The root idea is not a part of the tree idea or the bush idea but clearly, there is some correspondence between them. What exactly is the nature of this correspondence? Maybe it can be reduced to the problem of participation: when a concrete tree participates in the tree idea, then this tree must have a root which participates in the root idea. In this way, an indirect correspondence can be found between the root idea and the tree idea whereby the role of the empirical world is greatly increased because it allows for establishing such a correspondence between eternal ideas. Similarly, there exists only one unit idea to which all the number ideas are related, but there is no need to include a unit in the latter ideas to make them real (eternally existing). In Zermelo-Fraenkel-Skolem's set theory, number 1 is defined as the cardinality of the set $\{\emptyset\}$, where $\emptyset$ is an empty set, number two as the cardinality of the set $\{\emptyset, \{\emptyset\}\}$, number three as the cardinality of the set $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$, etc., but there is only one empty set $\emptyset$ and in all the sets used to define numbers there is a reference to the same set $\emptyset$, not to two of its copies, to four of its copies, etc.[10]

---

[6] Robert L. Patterson, *Plato on immortality*, University Park: The Pennsylvania State University Press 1965, 124.

[7] Léon Robin represents the most extreme case because he reconstructed Plato's theory of numbers in his *La théorie platonicienne des idées et des nombres d'après Aristote* (1908) by making no references to Plato's writings at all.

[8] Anders Wedberg, *Plato's philosophy of mathematics*, Stockholm: Almquist & Wiksell 1955, ch. 5; Gottfried Martin, *Klassische Ontologie der Zahl*, Köln: Kölner Universitäts Verlag 1956, 26. Pritchard is leaning in this direction, Paul Pritchard, *Plato's philosophy of mathematics*, Sankt Augustin: Academia Verlag 1995, 156, but see p. 94. After stating that "the connection between the number and the idea is one of the most unpleasant difficulties in all of the Platonic question" Cohen concluded that "Aristotle heard it correctly: that namely the mathematicals are positioned between the perceptible and the ideas," Hermann Cohen, *Platons Indeenlehre und die Mathematik*, Marburg: Pfeil 1878, 17, 31. Cf. an interpretation which sees in Plato three classes of objects: the forms (being), images that include immutable dianoetic copies and mutable sensory copies of the forms, and images of images (nonbeing). Numbers (and mathematicals) can be divided into these three types, Jolanta Świderek, *Rozważania matematyczne w pismach Platona*, Lublin: Wydawnictwo Uniwersytetu Marii Curie-Skłodowskiej 2002, 64-66, 84. Soul and mathematical entities considered to be intermediate beings between the intelligible and the sensible worlds by Cornelia J. de Vogel, *Philosophia, part I, Studies in Greek philosophy*, Assen: Van Gorcum 1970, 191.

[9] Plato could very well agree at that point with Locke that the idea of a triangle "must be neither oblique nor rectangle, neither equilateral, equicrural, nor scalene; but all and none of these at once" (*Essay* 4.7.9), so that a spatial element is absent in such a characterization. Thus, such an idea is not as impossible – as ridiculed by Berkeley (*Principle*, Intro., §13) – as it may appear.

It is also not necessary to assume that there are infinitely many copies of the same number.[11] Such an assumption may be motivated by using the same number more than once in one equation. Do we use the idea of the number three twice in the equation $3 \cdot 3 = 9$?[12] However, would we even ask such a question if the equation were $y = x \cdot x$? It is obvious now that in the last equation there are two references to the same variable x, not that this variable is used twice, or, worse yet, two copies of the same variable. It is only a syntactical requirement that x occurs in the equation twice, where x is the variable name, but not the variable itself.[13] Similarly, in the equation $3 \cdot 3 = 9$, the digit 3 is used twice, not the number idea 3. The equation is our, somewhat limited, way of expressing an eternal truth, but it does not mean that we, as it were, create multiple copies of ideas only because we refer to the ideas more than once.

So it seems that it is an Aristotelian anachronism to impose on Plato a tripartite division of numbers and seeing in his writings another layer of reality, the world of mathematical numbers between the world of perception and the world of ideas, only because such a division can be found in the *Metaphysics*. There are two types of numbers, physical numbers and ideal numbers. Ideal numbers are ideas because they are immutable as all ideas (*Rep.* 527b) and the same intellectual intuition (*noesis*) captures numbers and ideas (525a,c).[14] The Aristotelian concept of the mathematical number is alien to Plato.[15]

Regardless of the precise reconstruction of the concept of number, there is no doubt that for Plato there exists an infinity of mathematical entities: "powers [squares] are unlimited in number" (*Theaet.* 147d); number is "unlimited in multitude" (*Parm.* 144a). And because the number of forms and numbers is infinite and it takes an infinite mind to comprehend all of them, which is needed to create a perfect world, the Demiurge's comprehension ability must be infinite. Even creating the best finite world requires an infinite knowledge because the order is definable only in numerical terms as proportion and harmony. Even if the universe were limited in time, the Demiurge would have to possess infinite cognitive capacity to choose from an infinity of possibilities the one which is best. God made the world by bringing chaos into order, that is, by fashioning with "forms and numbers" what was "without proportion and measure" (*Tim.* 53ab).[16] The amount of forms and numbers is infinite and this requires an infinite mind to consider them. The order of the universe is thus based on the order inherent in the Demiurge, and the Demiurge's knowledge of this order. Even creating the best finite world requires an infinite knowledge because the order is definable only in numerical terms as proportion and harmony, and the amount of numbers is infinite.[17] The order then presupposes infinity. The *Timaeus* does not make it explicit, so to see it better, we turn now to the *Philebus*.

10.3 INFINITY IN THE *PHILEBUS*

The *Philebus* is concerned with the problem of pleasure and its role in a happy, well-rounded life vs. the role of knowledge and truth. Parenthetically, Plato discusses four elements of reality: the *apeiron* (the unlimited), *peras* (the limit), mixture, and cause. The discussion is very sketchy and not always unequivocal.

The *apeiron* forms a class of entities because the *apeiron* is a unity in that it includes entities of the same kind, and this class is "marked out by the common character of the more or less" (26cd). Whatever can become "more and less" or "too much" is of the genus of the unlimited (25a). The elements of the *apeiron* are characterized by fluidity, by admitting of degrees, by being indefinite. These can only be properties or qualities, such as the heat, longitude, temperature, color, pleasure, of which there can be more in one case and less in another. Plato explicitly mentioned pleasure and pain as elements of the class *apeiron*: "both pleasure and pain admit the more and less and belong to the unlimited kind" (41d). It may appear then that the *apeiron* is composed of qualities. However, we also read that the hotter and the colder, the more and less "always go on do not stand still," whereby "they turn out to be unlimited" (24d), that is, the hotter and the colder, the ability of temperature to have gradations, is included in the *apeiron*. Also, Plato listed "the high and the low, the fast and the slow" among the elements of the *apeiron* (26a). It may appear now that qualities of qualities are included in the *apeiron* as well. This, however, would create an nonhomogenous class: the class of qualities and qualities of qualities. Moreover, there would not be any connection between temperature and the hotter and the colder in the *apeiron*. It seems that Plato used different ways of saying that the *apeiron* is composed of elements (qualities) seen from the perspective of their having degrees.

---

[10] It is an interesting proposition that ideal numbers are collections of ideas, Jacob Klein, *Greek mathematical thought and the origin of algebra*, Cambridge: The MIT Press 1968, 89-90. This, however, raises the problem of the hierarchy of numbers in the spirit of Russell's theory of types, or the third man problem: what to do with the numbers themselves, can they be elements of numerical collections? If yes, what to do with these higher level collections? Can they be elements of other collections? Etc.

[11] As assumed by Wedberg, *op. cit.*, 77: "the mathematical number 2 … is not a unique something, since there are infinitely many mathematical 2's."

[12] The question is asked and answered in negative by Gottfried Martin, *op. cit.*, 23.

[13] This is even clearer if $y = x \cdot x$ is considered as an assignment statement in a programming language (e.g., `y = x * x`, as in Java), because x is a reference to the content of a memory location, and in this assignment, we refer twice to this content.

[14] See also G. Rodier, Les mathématiques et la dialectique dans le système de Platon, *Archiv für Geschichte der Philosophie* 15 (1902), 481.

[15] The famous analogy of the divided line at the end of the sixth chapter of the *Republic* does not imply that mathematical entities are of different nature than ideas. The difference between *dianoia* and *noesis* lies in the reasoning process including substantiation of premises (hypotheses). After all, *dianoia* as *dia-noesis* is just a lesser kind of *noesis* because it uses physical images of ideal mathematical entities, not because these entities are not ideas. See also Gaston Milhaud, *Les philosophes-géomètres de la Grèce. Platon et ses prédécesseurs*, Paris: Alcan 1900, 267, 274-276.

[16] We can thus agree with the statement that the Demiurge is not only the perfection, but infinite perfection, Jules Simon, *Études sur la théodicée de Platon et d'Aristote*, Paris: Joubert 1840, 202.

[17] "If there is number, there would be … an unlimited multitude of beings" because each number is also a being (*Parm.* 144a).

Elements of the class *apeiron*, such as pleasure, possess "neither beginning, nor middle, nor end" (31a). Pleasure admits of different degrees and as such it is *apeiron*. The continuum of pleasures along with the possibility of saying about any two pleasures that one is lesser than another becomes an element of the *apeiron*. This continuum has no beginning, middle, or end, thus it is infinite. There is an infinity of pleasures, but any two of them, as elements of the same continuum, can be meaningfully compared. Plato was more interested in that the elements or the *apeiron* admit of degrees and have to be defined in each particular case than in that these elements are infinite. The infinity of these continua is a side-effect of their indefiniteness. The qualities in the class of the *apeiron* are indefinite because they may have no bounds, have no beginning or end; they may also be indefinite because they have bounds that cannot be reached and they admit of an infinite number of degrees; they are endless in the number of degrees.

There are attempts to include in the class *apeiron* not only continua, but also elements of continua. For example, the statement that elements of the class *apeiron* possess "neither beginning, nor middle, nor end" is interpreted to mean that "we cannot meaningfully talk about a beginning, middle, and end" of "an object that has no specific measure."[18] However, it does not seem possible that an object has no specific measure, e.g., no specific temperature or mass. The measure can be unknown to us, but the object still has one and as such, the measure is defined if only in itself. But even if we know the measure, what would it mean to "meaningfully talk about a beginning, middle, and end" of an object or its measure? Moreover, the statement that "pleasure and pain admit the more and less and belong to the unlimited kind" (41d) does not imply that "individual pleasures and pains" can also be characterized as *apeiron*[19]; this is supposedly so because Plato was talking about comparison of pains and pleasures which is meaningful only when particular pains and pleasures (pains and pleasures of particular magnitude) are compared; but although comparing continua may be reduced to a comparison of their elements, this does not mean that the elements are of the same genus as the continua.

The second class discussed in the *Philebus* is the class *peras*, the limit. Whatever can be determined as equal or double or anything "that is related as number to number or measure to measure," belongs to the category of the *peras* (25ab). The class *peras* is made out of what is "commensurate and harmonious" because a "definite number" is imposed on them (25e). For instance, in frost and heat, limit "takes away their excesses and unlimitedness, and establishes moderation along with harmony" (26a), so it appear that "frost and heat" are *apeiron* if characterized as excessive and *peras* if seen as harmonious. Frost and heat are temperatures, opposite sides of temperature, so that temperature may be characterized at one time as *apeiron*, at another as *peras*. Moreover, the "excess and the overabundance" of our pleasures is contrasted with "law and order" imposed as a limit on pleasures (26b). This indicates that pleasures, like temperature, seem to belong both to the category of the *apeiron* and that of the *peras*. How can this be?

The *apeiron* is a set of qualities characterized qualitatively. We may say that one object has quality Q to a lesser extent than another object. All the objects can be ordered by a relation proper to Q. Numbers are not involved in the characterization of the elements of the *apeiron*, although this can be done. We can map, for example, all temperatures and the relation of hotter-and-colder onto the domain of real numbers and the relation <. But this is not the concern of the set *apeiron*. Qualitative characterization of qualities is all that is needed to determine the elements of this set. The qualities in the *apeiron* usually have an infinite range and so, numerically, the ranges are the same in size, but, materially, the ranges are different. Each element in the category *apeiron* is thus a pair (quality Q, $<_Q$), for example, (temperature, the hotter and the colder) or simply (temperature, $<_{temp}$); (speed, the fast and the slow) or (speed, being slower than) or simply (speed, $<_{speed}$); (pleasures, excess) or (pleasures, being lesser than); etc.[20] Particular qualities Q along with the corresponding relations $<_Q$ are particular substrata included in the set *apeiron*, and in that sense they are raw material out of which particulars are formed. Temperature in that sense is a continuum, that is, a set of particular temperatures organized by the relation "being lesser than." This temperature is *apeiron* because it is indiscriminate about how, where, and to what extent is should be utilized. There are no landmarks on this continuum which would allow us to say what is too much, too excessive, too abundant. Such landmarks have to be added to it – and then temperature becomes *peras*, becomes tamed, friendly, usable, meaningful.[21] These landmarks are given as the laws of harmony imposed on each quality separately, so that each quality has laws specific to it, because harmony and proportion varies from one quality to another. Therefore, the *peras* is composed out of the same qualities as the *apeiron*, but instead of looking at them as uniform continua as the *apeiron* does, the *peras* takes these qualities along with the laws specifying their harmony and proportion. The latter, however, can only be defined with numbers, thus the *peras* characterizes qualities quantitatively.[22] In this way, the qualitative indeterminacy of qualities is overcome by making them a subject of quantitative laws. The association of the qualitative order of qualities with their quantitative order is not accidental because the art of measurement, "the art of the excess and deficiency" of one magnitude in relation to another in a word, can be accomplished by "nothing other than arithmetic" (*Prot.* 357a).

---

[18] Gisela Striker, *Peras und apeiron: das Problem der Formen in Platons* Philebos, Göttingen: Vandenhoeck & Ruprecht 1970, 41.

[19] Striker, *op. cit.*, 50.

[20] This seems also to be the meaning of interpretation which states that the *apeiron* should be distinguished from specific *apeira*: the former is "more and less," which is manifested in particular instances, such as "hotter," "colder," "strongly," "softly," "calmly," etc., Sylvain Delcomminette, *Le* Philèbe *de Platon*, Brill: Leiden 2006, 226.

[21] For Plato, the unlimited is worthless until "it receives through the limit an order and thereby some value," Jonas Cohn, *Geschichte des Unendlichkeitsproblems im abendländischen Denken bis Kant*, Leipzig 1896 [reprint Hildesheim: Georg Olms 1960], 34.

This discussion can be summarized in the statements that it is not the case that

apeiron = {height, speed, …, quality Q, … }

and not

peras = { height$_1$(object$_1$), height$_1$(object$_2$), … height$_i$(object$_1$),
height$_i$(object$_2$), …, speed$_1$(object$_1$), … Q(object$_k$), ..}

but instead (the names of relations are Plato's)

apeiron = {(height, the high and the low), (speed, the fast and the slow),
(temperature, the hotter and the colder),…,
(quality Q, more-or-less$_Q$ (also called excess-and-overabundance$_Q$
or excess-and-limitedness$_Q$)), … }

and

peras = { (height, law-and-order$_{\text{height}}$), (speed, law-and-order$_{\text{speed}}$), …,
(quality Q, law-and-order$_Q$ or moderation-and-harmony$_Q$), … }

Existing things are generated from the mixture of the *apeiron* and the *peras* (*Philebus* 16d, 27b). This is a mixture in the sense that everything that is, is characterized by many qualities and these qualities have to be in proper proportion. For example, pleasure and pain by themselves belong to the *apeiron* (41d), but lamentation is a mixture of pleasure and pain, "and [so it is] on myriads other" occasions (50b). Similarly, man is a being characterized by height, temperature, movements with a certain speed, etc. If any of these qualities is exceeded, then an improper being arises, an unfit being. There cannot be a man 20 feet tall. Such a mixture is inadmissible; in fact, it is not a mixture at all. However, a 20 foot height is proper for trees and a tree mixture that includes the height of 20 feet is admissible. Proportions between height, temperature, speed, etc. that characterize man are different than proportions between these qualities characterizing a tree. "Any mixture that does not one way or another possess measure or the nature of proportion will necessarily damage its ingredients and primarily itself; for there is no mixing in such cases at all but really some unmixed assortment" (64de).

To create a proper mixture, knowledge of numbers is necessary. "If someone were to take away counting, measuring, and weighing from all the arts [including the art of creating mixtures], the rest might be said to become worthless" (55e). Knowledge of numbers is a necessary prerequisite in creating the laws for particular qualities. The Demiurge can impose proper laws and order only if endowed

---

[22] It is a simplification to state that the *peras* encompasses all numbers and measures but no laws: Striker, *op. cit.*, 60, 61, 68. The numbers and measures have to be meaningfully associated with qualities and this is done by laws corresponding to the qualities. It is then incorrect to exclude these laws from the *peras* and retain only numbers isolated from the qualities.

with the knowledge of proportion and measure that is built, among other things, from the knowledge of numbers. This knowledge is also necessary to create mixtures to mix ingredients in proper proportions. This knowledge thus has to be infinite if only because the amount of numbers is infinite. Infinity therefore is present not only in the *apeiron* as an infinite extension of qualities constituting the *apeiron*, but also in the *peras* to create proper laws and order for particular qualities, and also in the mixture, to mix the qualities in proper proportions. The *apeiron* indicates a passive presence of infinity in the form of infinite ranges of qualities to be operated on. The *peras* and the mixture indicate an active presence of infinity in the form of knowledge brought into motion by the cause, the fourth kind of reality in the *Philebus*, to establish proper laws for elements of reality and then to put these elements together in a proper way. In both latter cases (establishing laws and combining qualities) the cause-Demiurge may be assumed to use the ideas as the model. These ideas are not explicitly mentioned in the *Philebus*, but by using them, the cause-Demiurge can mold the *peras* and the mixture with the infinite cognitive capability.

10.4 THE *PHILEBUS* AND THE *TIMAEUS*

There were many attempts to find a correspondence between the quadruple from the *Philebus* – the *apeiron*, *peras*, mixture, and cause – and the quadruple from the *Timaeus* – the Demiurge, being, space, and becoming – none of them successful, if a direct, one-to-one correspondence was attempted. The foregoing discussion suggests the following solution. There is only one direct correspondence, namely the Demiurge corresponds to the cause. The cause described as "our king of heaven and earth" (*Philebus* 28c), an all-encompassing wisdom (30b), wisdom and reason (30c), is clearly a counterpart of the Demiurge.[23]

The being (ideas) is only implicitly present as the model used by the cause to form the *apeiron*, *peras*, and mixture.[24] These three kinds were produced by the craftsman (27ab) and as such they cannot have any direct counterparts among the four kinds of the *Timaeus* which are all eternal. The *apeiron*, *peras*, and mixture were produced by the cause which used the ideas as models for producing properties and their ordering relations and then the laws of their harmony (such ideas as the equal and the greater and the smaller (*Phaedo* 75c) were certainly instrumental in this process). Therefore, in the statement that "whatever things are said to be consist of one and many, having in their nature limit and unlimitedness" (*Philebus* 16c), the phrase "whatever is said to be" cannot refer to anything eternal because the *apeiron* and the *peras* are not eternal – they are produced. The "whatever things are said to be" refers to the created world, to our universe in which everything has the *apeiron* and the *peras* in its nature, because everything in this world is a mixture of the two kinds.[25]

---

[23] The identification of the Demiurge with *nous* is discussed by Stephen Menn, *Plato on God and nous*, Carbondale: Southern Illinois University Press 1995, 7-10.

[24] It does not seem then to be correct to see the ideas to be the result of the activity of the cause in the *Philebus* as it is sometimes assumed, e.g., Léon Robin, see Leo Sweeney, *Divine infinity in Greek and Medieval thought*, New York: Peter Lang 1992, 108. Some identified ideas with the *peras*, e.g., Cornelia J. de Vogel, La théorie de l'apeiron chez Platon et dans la tradition platonicienne, *Revue philosophique de la France et de l'étranger* 84 (1959), 22, 25. See also a discussion in David Ross, *Plato's theory of ideas*, Oxford: Clarendon Press 1953, 133-136.

Space is not used at all in the *Philebus*. However, we can interpret its indirect presence in at least one instance. Space is characterized as "devoid of any characteristics" (*Tim.* 50e, 51a), therefore, the movements of matter in space before the world was formed were truly chaotic; there was no up or down, there was no order in space. This qualitative ordering of space had to be added, to distinguish upward and downward movements, to talk meaningfully of a particular direction. This was done by creating the qualitative order and saturating space with it, that is, by adding directedness to space. Space is then only qualitatively characterized; different directions can be distinguished and motions can be qualitatively ordered.

The prime matter, the realm of becoming, is molded first by endowing it with properties, i.e., by saturating it with the *apeiron*. The matter became capable of having temperature and the temperatures could be ordered. The matter became capable of displaying colors and colors could also be ordered. The matter became saturated with properties that later are put in different configurations and intensities in mixtures.

The *peras* specifies the laws proper to particular qualities that can now be introduced in the matter. At first, the matter was "without proportion and measure" which have to be infused into matter as laws or quantitative order and harmony, as laws phrased in terms of "forms and numbers" (*Tim.* 53ab). In this way the stage is set, and in this way we can interpret the statement that "before the coming of time, the universe had already been made to resemble in various respects its model" in whose likeness the Demiurge was making it (39e). Before creating every body and soul in the universe, the cause-Demiurge prepared qualities with which the bodies and souls were formed. The raw prime matter is tamed qualitatively with the *apeiron* and then quantitatively with the *peras*. In this way the cause-Demiurge persuades the necessity reigning over matter to use these elements, in particular, the laws from the *peras*, to execute its movements, and to make these movements in matter more regular, more orderly. These laws and the ideal models are used by the cause-Demiurge to create particular beings in the universe, both bodies and souls, in a word, to create a mixture. The result is the world.

10.5 TIME AND ETERNITY

The result of molding prime matter by the Demiurge in accordance to the eternal paradigms is our world, the only world the Demiurge created. This spherical

---

[25] Usually "whatever is said to be" is identified with everything that exists, including the world of ideas. An extreme view is to identify it only with ideas, as done by Striker, *op. cit.*, 22; cf. discussion in Justin C.B. Gosling, *Plato's Philebus*, Oxford: Oxford University Press 1975, 84.

world brought into rotational motion and endowed with a soul is as perfect as a world can be, thus being itself "a blessed god" (*Tim.* 34b). However, one thing was missing in this perfection. As created, the universe was not eternal. It did not have an infinite duration. The Demiurge, the ideas used as models for the universe, and primal matter are truly eternal because they are everlasting entities about which it can be simply said that they *are*. The universe *is* only in a qualified sense because its being has a starting point and its being was initiated by and owes its existence to the Demiurge. As such, the world apparently cannot share the Demiurge's eternity. This eternity can only be imparted on it in the form of never-ending time which embraces it, and within which it can only be said that the universe is because each point of time is associated with a state of the universe. Time only imitates eternity; it is only a "moving image of eternity" (37d) whose sole function is to make the universe close to perfection. But how is it possible that creation includes creation of time, and yet motion of space was caused by necessity? Does it not involve a contradiction? The Demiurge and ideas are immovable in their eternal respite, but primal mass is in motion; therefore, it is possible to distinguish a before and an after of each point of prime matter based on its positions in space at the beginning and at the end of some movement. In the Aristotelian fashion then, time can be defined as a measure or as a number of motion (*Phys.* 223a33). The contradiction can be solved by observing that the Aristotelian definition of time refers to orderly, periodic movements, so that in the absence of uniform movement there is no time. And this is certainly a case with unordered movements of prime matter in the pre-cosmic space. Although events taking place in this space can be put in a sequence, the sequence is of a nontemporal nature because the events are not ruled by any regular law. Time begins with the formation by the Demiurge of earth, moon, sun, planets, and fixed stars – and their movements.[26] When movements become regular, that is, periodic, time comes into being. Time, therefore, is characterized by periodicity, by cyclic character of days, weeks, and years. Time, in that limited sense is cyclic; limited – since recurrence of nights and days, summers and winters does not mean repetition of events.[27] Time is simply an imposition of regularity on these events which themselves do not have to recur; they only take place in a structured framework.

Through an imposition of orderliness onto things, time mimics eternity by reflecting its permanence and relative immutability. Periodicity of motion of celestial bodies is immutable in the sense of repetition of motion, although motion itself is a change. Thus, time shares stability and permanence with eternity. Ideas are eternal since what is eternal "is always immovably in the same state" (*Tim.* 38a) and so are ideas: immovable, always the same (28a, 37b) and thus can serve as paradigms for particular entities. The Demiurge is eternal (34a, 37a) since he does not change in his timeless existence. When this permanence of temporal evens becomes infinite, time becomes truly an image of eternity. And so we have eternity as infinite duration and Platonic time and temporal infinity as infinite duration punctuated by the planetary metronome.

---

[26] "It is only when the regular motion of the heavenly bodies comes into being that time begins," Gregory Vlastos, The disorderly motion in the *Timaeus*, in R.E. Allen (ed.), *Studies in Plato's metaphysics*, New York: Routledge & Kegan Paul 1965, 387; see also his distinction between uniform time, time(U), and irreversible temporal succession, time(S), Creation in the *Timaeus*: is it a fiction?, in R.E. Allen (ed.), *op. cit.*, 410-411.

[27] "The recurrence or identity in kind of the units of periodic repetition, such as day, summer, or year, can very well go together with numerically different events," W. von Leyden, Time, number, and eternity in Plato and Aristotle, *Philosophical Quarterly* 14 (1964), 43. Time is "the image of eternity moving according to number … the πέρας imposed upon an ἄπειρον of duration," J[oseph] B. Skemp, *The theory of motion in Plato's later dialogues*, Cambridge: The University Press 1942, 111.

10.6 KNOWING THE INFINITE

The knowledge of the infinite is not only possible but also necessary for the rounded education of the Platonic city and mathematics is a means of actualizing this knowledge. Mathematics is the means of overcoming the limitations of sensory perception which can be deceptive. Something which is large appears to be small from a distance (*Prot.* 356c, *Rep.* 602c). The change of size is in the eye of the beholder only. If such apparent changes also take place in the realm of ethics, then we are doomed to relativism, to changing our view to what is good or bad according to the current situation. Such a situation ethics is not a viable alternative to Plato. True knowledge and true ethics have to be anchored in the absolute, immutable, in what can be a reliable guide in all situations. This absolute can be found only outside the world of the perceptible and mathematics is a sure guide to this realm. So Plato said that the confusion of the perceptible appearance can be lifted if we refer to the art of measurement, the art of relative excess and deficiency of magnitudes, which is made useful by "nothing other than arithmetic" (*Prot.* 357a, *Rep.* 602d). This principle is not motivated by scientism, which became a hallmark of philosophy starting from the age of the Enlightenment, not motivated by the desire to reduce philosophical, ethical, and theological issues to the science of numbers, but by the desire to make science in general a repository of all meaningful answers. Plato did not by any means intend to categorize the philosophical problems as pseudoproblems, but he wanted to bring to the fore the extrasensory sphere, to root the true knowledge in what exceeds the realm of the natural world, to look for the truth in the world of the absolute. By being a means of taming the appearance in the sensory world, the art of measurement points to arithmetic and thereby to numbers as the means of acquiring true knowledge. Because numbers are in the extrasensory sphere, mathematics leads the truth seeker to the world of ideas, to the world of the absolute.

In Plato's world, there is a clear priority of the extrasensory sphere over the sensory world both in the ontological and epistemological order. Not only has the universe a beginning and the world of ideas does not, but the knowledge of the world is only possible if we possess the knowledge of the ideas. In particular, numbers are not constructed in an empiricist fashion by using the data filtered though perception. The soul considers numbers and relations between numbers "alone and through itself" and not "through the bodily powers" (*Theaet.* 185e).

These data, at best, give an opportunity to bring to the foreground images of ideal numbers stored in the hidden recesses of the mind. These data can be truly understood in terms of knowledge about the ideas and about numbers. In cognition, we really descend from the world surpassing the universe of ours to the level of perception and only in this way perception can be organized into a cognitively meaningful whole. This is an approach which only after centuries becomes reality in science by mathematization of natural sciences, and it is true that "Galileo and Kepler ... could in the restructuring of method of modern natural sciences directly and consciously refer to Plato."[28] Mathematics is a conceptual web which allows us to capture and to organize the data and the world. Mathematics is a theoretical prerequisite of true knowledge of the world accessible to the senses. Empirical knowledge is possible through mathematics. Mathematics, on the other hand can well exist without the empirical knowledge.

Mathematics in general, the art of calculation and thus the knowledge of numbers in particular, is an indispensable part of education for its own sake and for the sake of the truth to which it leads. People who are "naturally skilled at reasoning are naturally sharp ... in all subjects" (*Rep.* 526b). Therefore, to think that astronomy and mathematics are unnecessary is downright stupid (*Laws* 818cd). By mastering mathematics people define their very humanity. It is necessary for a warrior to be "able to count and calculate" not only to set his troops, but "even to be a human being" (*Rep.* 522e).[29] How can that be? The Demiurge, an infinite being, can quite naturally comprehend the knowledge objectively existing in the world of ideas. But humans? They are finite beings and yet Plato says that someone who has completely mastered arithmetic "knows all numbers because there is knowledge of all numbers in his soul" (*Theaet.* 198b). Partial knowledge of arithmetic means knowledge of some numbers. This may be sufficient to have some knowledge of the world of ideas. But knowing all numbers means knowing an infinity of them. This may mean naming them one by one, which is rather unlikely although not impossible considering that the soul is immortal so that its duration is not a limiting factor of such a feat. However, even allowing for a possibility of naming all the numbers, they have to be known before they can be named. Thus, although we are finite beings, we are bearers of actual infinity, even if it is only the infinity of numbers. The knowledge of numbers is already inscribed in our mind. If it were not, the naming process would stop: How one can talk about something truly original, something which is not a reflection of an eternal idea? The mind is thus pregnant with infinity, although this infinity is not readily available, and it may take a painstaking education process to unravel this infinity. But it is already there, and we have to find our way to it. Human being is finite, yet infused with infinity.[30]

---

[28] Seth Demel, *Platons Verhältnis zur Mathematik*, Leipzig: Meiner 1929, 69.

[29] A comment ascribed to Plato says that the superiority of man consists in "that he alone of all animals knows how to count", Aristotle, *Problemata* 956a11.

[30] It is thus a major misconception in the statement that "Plato was afraid of the infinite," Peter Gorman, *Pythagoras: a life*, London: Routledge & Kegan Paul 1979, 140.

There are means of harnessing this infinity of numbers even in our earthly life full of limitations. Such means include the systematic way of deriving new numbers with a recursive definition from the knowledge of the already known numbers.[31] This may also mean proving theorems about infinity of numbers in the finite numbers of steps, and the principle of mathematical induction can be such a means.[32] In any event, recursive definitions and mathematical induction are the means of capturing infinity, conquering it so that man, a finite being, can, in a way, encompass it in his mind. This is not an impossible task, although it can be perfectly mastered by but a few, because it is an arduous task and it would be difficult to find subjects that "require more effort from the one who learns and practices [them] … than this" (*Rep.* 526c). But the one who is capable of completely mastering arithmetic not only is closer to the world of ideas, but he also masters infinity. He sees its presence not only in the infinity of numbers but also in the infinity of ideas and in the infinity of the truth and the good. He realizes that the universe is founded not only on the absolute character of the world of ideas but also on infinity of this world, on infinite knowledge of the Demiurge and an infinite duration of the four kinds of reality: the Demiurge, world of ideas, prime matter, and space. He realizes that although the created world is finite in space, it is the work of the infinite being, whose infinity is reflected in time, "the moving image of eternity," in time, whose infinity can be understood with the aid of the infinity of numbers.

---

[31] An example of such a definition is shown in the *Parmenides* 143c-144a where after assuming that 1 exists, 2 is inferred, 3 is defined as $1 + 2$ and then a sequence constructed: $2 \cdot 1$, $3 \cdot 1$, $2 \cdot 2$, $3 \cdot 3$, $2 \cdot 3$, $3 \cdot 2$. The four last terms of this sequence show that there exist even times even, odd times odd, even times odd, and odd times even which proved to Plato that there is an infinity of numbers. The proof itself may be generalized to generate numbers $2n$, $3n$, and $2n \cdot 3m$ (Wedberg, *op. cit.*, 140) so that prime numbers are left out. The definition, however, allows for generating an infinity of numbers.

[32] An incomplete version of a proof by induction can be seen in Plato's proof that for n items put in a row, the number of contacts between them is equal to $n - 1$ (*Parm.* 149a-c).

11 ARISTOTLE

Aristotle discussed the problem of infinity and devoted much more attention to it than his predecessors; however, his treatment of infinity and his assumptions concerning its existence are largely at variance with the way Greek philosophy treated infinity before him.

No predecessor of Aristotle gave a definition of infinity, but their understanding of this concept can be gleaned from the way they applied it in their discussions. Usually, in the spirit of mythology, temporal infinity was taken for granted, and in most cases spatial infinity was assumed to exist – in the form of an infinite space, or void, or universe. The concept of infinity reached its pinnacle in Anaxagoras' view that everything can be extended indefinitely and everything can be indefinitely divided; this extendibility and divisibility was a reality to Anaxagoras, not just a possibility. A great change came with Aristotle who, probably because of Zeno's paradoxes, became overcautious to the extent that "the definition of infinity as used in Aristotle must be considered a step backward if we compare it to Anaxagoras' theory"[1] so much so that "the problem of infinity ... is one of the most embarrassing [problems] in Aristotle's philosophy."[2]

11.1 DEFINING INFINITY

Aristotle's troubles with infinity started with its definition. He defined infinity as that which cannot be traversed since it is not "its nature" to be traversed, and any attempt to traverse it is guaranteed to have no termination (*Phys.* 204a3-7). By definition then, infinity is an unlimited overcoming of limits with no prospect of finishing the process. Infinity cannot, therefore, be seen as something complete, only as something in the process of unfolding. Infinity is bestowed with meaning by a process of manufacturing infinity; that is, infinity is not an entity to which the process is applied. However, infinity is not produced *ex nihilo*, and an object has to exist in order for the process to be carried out, and an example of such an object could be an interval to which the process of bisection can be applied. However, such a process can never be completely actualized; it manifests itself only through partial accomplishments. The infinite can only be seen through the finite, the latter being the only form of existence of the former. Therefore, "it is not what has nothing outside it that is infinite, but what always has something outside it" (*Phys.* 207a1-2).[3] In a trivial interpretation, everything except for the universe would be infinite, since everything has something outside of it. In a non-trivial interpretation, we can have a contradiction, because there may not be any "what" which can be predicated as infinite: if this "what" is understood as a set which can always be expanded by adding more elements, then, by definition, it never exists, and if it can be considered existing, it is only finite, hence not infinite. Therefore, a definition of infinity has been constructed in such a way that infinity cannot be brought into being, and it is just an analytical truth that it is unrealizable.

---

[1] Theo G. Sinnige, *Matter and infinity in the Presocratic schools and Plato*, Assen: Van Gorkum 1971, 135.

[2] Léon Robin, *Aristote*, Paris: PUF 1944 [reprint, New York: Arno Press 1979], 144.

[3] For discussion of this important passage, see Abraham Edel, *Aristotle's theory of the infinite*, New York: Columbia University Press 1934, 57-58.

Aristotle also made a distinction between infinity by addition and infinity by division (*Phys.* 204a7), and because "nothing is complete which has no end/limit" (*Phys.* 207a14), infinity is an imperfect state and thus less desirable. This state cannot, however, be actualized: infinity by addition does not exist in any sense; infinity by division exists only potentially with no prospect of becoming real.[4]

11.2 ARISTOTLE'S RAZOR

In the light of this brief discussion, Aristotle's attitude toward infinity should not be surprising. His position with respect to infinity can first be seen in a parenthetical remark he made concerning the use of the concept of infinity in explanatory premises – choose, if possible, the finite rather than infinite: "when the consequences of either assumption are the same, the limited should be chosen rather [than the infinite], since in things [constituted] by nature that which is limited and better should, if possible, be present rather [than the opposite]" (*Phys.* 259a9-11).

Aristotle's analyses indicate that this remark should be strengthened: not only is explanation in terms of the finite always easier and better than in terms of the infinite, but infinity in all respects should be a suspect concept and avoided even at the cost of making explanations more complex. This principle can be called Aristotle's razor, to be used to remove infinity from theory and from nature. This razor is a primary principle used by Aristotle in seeing the world as finite and in seeing it through finite means. Aristotle was not altogether consistent in applying this razor, but it was one of the most powerful tools he used in constructing his impressive system, especially in physics.

It is noteworthy that Aristotle's razor is not just a form of Occam's razor,[5] since it specifically singles out infinity as a worse assumption than finitude. In fact, these razors can be applied independently and examples can be found in Aristotle himself. He applied Occam's razor when he said that "to postulate pores is superfluous," since explanations can be made without them (*De gen. et corr.* 326b21); and after discussing the problem of the Unmoved Mover (UM), he concluded with a rhetorical question: "what need then is there to look for other principles?" (*Met.* 1072a18). He applied both razors when he dismissed Anaxagoras' explanations because they unnecessarily rely on the assertion concerning an infinite number of elements, "since the hypothesis of an infinite number will give identical results" (*De caelo* 302b21-23, similarly *Phys.* 189a13-17). In his treatment of time and its infinity, Aristotle suspended his razor but applied it to the causal chain and the possibility of its infinity, which resulted in the necessity of a prime mover, in spite of Occam's razor, which was later criticized by Theophrastus.

11.3 INFINITY BY ADDITION

Who is most qualified to scrutinize the question of infinity? According to Aristotle, "it is appropriate that the person who specializes in physics should investigate the infinite, whether or not it exists and if it does, what it is" (*Phys.* 202b34-36). Hence, Aristotle approached the problem of infinity as a physicist: the dictates of nature are the most important and observation is the main guide in making certain claims. Perception indicates that infinity is not a part of nature, that we are surrounded by finite objects, that things come into being and dissipate, and that our knowledge is limited. The dictates of perception indicate that there is a limit to the universe, and reasoning shows that there is no void, in particular no void outside the heavens, since there could be a body in it – an infinite body, or an infinity of bodies/worlds – since "in the case of eternal things what may be does not differ from what is" (*Phys.* 203b30) since what is eternal is necessary and what is necessary is eternal (*De gen. et corr.* 337b35-338a2) and "it may be" can be understood as "what is necessary is possible" (*Anal. pr.* 25a37-39).[6]

The infinity of substances would put their proportion out of balance; if, for instance, air would exist in an infinite amount and fire would be finite, so the properties of fire would be expunged by the properties of air (*Phys.* 204b11-19). Also, the presence of an infinite body would invalidate the concept of natural motion and natural place (205b24-35). Moreover, the universe is finite, since the infinite cannot make the smallest movement in finite time (*De caelo* 272b12-13), and one can see that the celestial bodies are moving. Also, a circular movement is first, and an infinite body would have to move in a circle of an infinite radius, and such a circle does not exist (271b27-272a7). Moreover, such a body would have to have an infinite weight, which is impossible (273b27-28). The infinite weight, in turn, is impossible since such a body would have to move with an infinite speed, because the heavier a body is, the faster it moves: "but there is no proportion between the infinite and the limited" (274a7-8).

---

[4] An interpretation of the cryptic phrase, "the matter of the completeness which belongs to size, and what is potentially a whole" (*Phys.* 207a21-22), concludes that "as matter is pure virtuality calling for form ... so infinity is a pure virtuality calling for the finite," Tony Lévy, *Figures de l'infini*, Paris: Seuil 1987, 45.

[5] Walter Bröcker, *Aristoteles*, Frankfurt/M.: Klostermann 1974, 275; George Boas, Presuppositions of Aristotle's physics, *American Journal of Philology* 57 (1936), 24-32, calls it an economy principle. Edel, *op. cit.*, 87, mentions a rule that "it is better where a finite number of principles explains sufficiently, not to invoke an infinite number."

[6] "Void is a place where body may be, and this 'may be,' under Aristotle's hand, turns at once into an 'is,'" Friedrich Solmsen, *Aristotle's system of physical world*, Ithaca: Cornell University Press 1960, 168.

This expulsion of infinity and the void from the universe is not without its paradoxes. One consequence of this view is that the universe is not located in a place: "outside the heaven there is and cannot be a body; it is thus clear that there is neither place, nor void, nor time outside it" (279a17-18).[7]

Thus, Aristotle's universe is of limited size and nothing can surpass this size. Therefore, if an indefinite increase in magnitude of objects – for example, line intervals – were possible, then "there would be something bigger than the heaven" (*Phys.* 207b20-21); hence infinity by addition is impossible because of the physical nature of the universe. Of course, the question could be asked why the universe could not expand indefinitely. Aristotle could counter such a possibility with an argument that an infinite amount of material would be needed, but whence would it come? If the universe were infinite and in self-motion, this would lead to an impossible situation since what is in self-motion is animate and an infinite animal is impossible (*De caelo* 275b25-27). In general, an infinite body, be it the universe, cannot move (272b12) since it would have an infinite weight (273a24-25) and thus could not move (274a3-10) and so is impossible (274a16).[8]

An immediate consequence of this limitation is that rectilinear movement is limited by the size of the universe. By moving in a straight line, we would eventually reach the first heaven, beyond which the movement cannot be continued – because no such beyond exists. There was, however, no limit to such movement in the cosmologies of Aristotle's predecessors, beginning with Anaximander and his grand vision of the universe emerging from the *Apeiron.* Aristotle became a key figure in the process started by Empedocles of transition from the infinite universe to the closed world which was reopened by the Neoplatonists and by Christian philosophy. It is quite ironic that Aristotle ruled out the possibility of an unhindered rectilinear movement, which in Newtonian mechanics became a type of movement requiring no support to remain permanent.[9] Since Aristotle's physics is built around his cosmology, the latter will have to be rejected to allow for the development of physics, and this development was possible only when an edge was taken off of Aristotle's razor.

Physics has an impact on the treatment of infinity in mathematics. However, by Aristotle's pronouncement, mathematicians really "neither need the infinite nor use it, but [they need] only that the limited [magnitude] be as large as they wish" (*Phys.* 207b29-31). This wish, however, has a limit which is imposed by the size of the universe: there can be no line longer than the diameter of the outermost celestial sphere (cf. 207b15-21). Hence, if two such lines can be produced, they cannot be joined together since their size would surpass this diameter. Therefore, from Aristotle's perspective, mathematicians' wishes should be confined to what is practically possible, and if these wishes contradict natural limits, they should be abandoned, according to the operation of Aristotle's razor. However, if mathematicians attempted to combine two such lines into one, what status would the resulting line have? It would not be and could not be an actual line, since natural limits would not permit such an actualization. Would it be potential? Not in Aristotle's world, since it has no chance to become actualized there. Is there thus a third state for such lines? They can be considered simply creations of the mind – not to say figments of the imagination – and included in a separate category, but they still would have some inward (mental) reality, although not outward. After all, if the world of ideas is denied existence, then where can all theoretical constructs of mathematicians be included if not among constructs of the mind? If these constructs are simply fantasies and absurdities, then mathematicians should not be considered seriously; but not even Aristotle would say that. Therefore, these mental constructs are possible and theoretically realizable. This realizability is not impossible, even from Aristotle's point of view. Aristotle made his pronouncements concerning infinity as a physicist, from the perspective of observation and perceptual experience, hence he defied infinity or restricted its limits as much as he could. But only hypothetically, he agreed that infinity might exist if it were somehow possible (e.g., if his God, the UM, were powerful enough to actualize infinity), since he said that if bisection could have been carried out indefinitely, "nothing impossible will have resulted if it [a body] has actually been divided – not even if it has been divided into innumerable parts, themselves divided innumerable times" (*De gen. et corr.* 316a21-23) and, in respect to infinity by addition, "it is always possible to take more than a given [magnitude] just as we say that number is infinite because there is no greatest [number]" (*De caelo* 271b33-272a2).[10] An infinite world is thus not impossible; it simply does not exist. Thus, infinite or just too long lines cannot exist in our world (the only world Aristotle knew), but they have some theoretical standing and hence a semi-real status as an adumbration of what could have been if the universe had been larger or even infinite. We can think about infinite lines just as much as we can think about unicorns, and thus they have some reality, if only in the mind. However, the physical reality prevents such mental constructs from achieving realization and thus they can never be actual entities.[11]

---

[7] An attempt can be made to make sense out of the view that, after all, the universe is somewhere, as stated by Benjamin Morison, *On location: Aristotle's concept of place*, Oxford: Clarendon Press 2002, 166-169.

[8] See also William Charlton, Aristotle's potential infinites, in L. Judson (ed.), *Aristotle's* Physics: *a collection of essays*, Oxford: Clarendon Press 1991, 131.

[9] Actually, Aristotle mentioned the possibility for an unhindered body to remain either at rest or in indefinite motion (*Phys.* 215a20-22), which was considered to be an adumbration of Newton's first law of motion by Thomas Heath, *Mathematics in Aristotle*, Oxford: Clarendon Press 1949, 115, 152, but this is mentioned in a chapter discussing the impossibility of the void, and infinite motion is given as an example of an absurd consequence of the view of the existence of the void.

[10] "By always adding to a finite [magnitude] I will exceed any given [magnitude]" (*Phys.* 266b2-3).

[11] One can speak about "potential actualization" in the mind of magnitudes greater than the size of the universe (Theokritos Kouremenos, *Aristotle on mathematical infinity*, Stuttgart: Steiner 1995, 57 note 105; cf. "potential actuality" in *Met.* 1048b16-17); however, they do not thereby acquire, so to speak, actual actualization. Therefore, speaking about "the sphere of the universe could be conceived as *actual* infinite for practical purposes" (p. 49 note 93) does not make the infinity actual; it still remains a mental construct.

The existence of infinity by addition may be defended by saying that the infinite by addition is an inverse of the infinite by division (*Phys.* 206b3). The steps of the bisection process can be numbered, and this number would grow indefinitely, but "this number is not separable from the process of bisection, and its infinity is not permanent but is in [the process of] coming to be, like time" (207b13-15). Therefore, the potentiality of one infinity enables the potentiality of another: they both would be equally real – or equally unreal. Hence, in their work mathematicians could replace arbitrarily large magnitudes by arbitrarily small ones (207b31-34). This, however, would not solve all problems since some geometrical assumptions require infinite extensions, e.g., the fifth postulate, and certain proofs may require some auxiliary constructions whose magnitude surpasses the size of the universe. A recourse to an abstract status of geometric constructs would not help, since "geometry investigates physical lines but not as physical" (*Phys.* 194a10-11), i.e., it utilizes physical lines as the point of departure, using only some of their attributes. Thus, such lines have to exist in order for the abstraction to be possible.[12] This means that mathematicians do more than just abstract from physical lines, otherwise the theory they build from what they observe in nature would not be Euclidian geometry, since Aristotelian cosmology is in contradiction with this geometry.[13]

11.4 CONTINUUM

There is no actual infinite, "but to suppose that the infinite does not exist in any way leads to many impossible consequences, that is clear. There will be a beginning and an end of time, magnitudes will not be divisible into magnitudes, number will not be infinite" (*Phys.* 206a9-12). Therefore, the problem of infinity cannot be avoided, and "the infinite appears first in the continuous" (200b17-18).

What is continuous? Things are "continuous when the touching limits of each become one and the same" (227a11-12). Continuous things cannot have indivisible parts, e.g., a line cannot be composed of indivisible points because "the extremities of two points can neither be one since of an indivisible there can be no extremity as distinct from some other part, nor the extremities are together" since a thing without parts has no extremity which would be its part (231a24-29). Therefore, the continuum cannot be reduced to the non-continuum, and it can be understood only in terms of dividing it into continua. In order to avoid the *circulus vitiosus*, the operation of dividing would be stressed rather than the result of this division, all the more that this division process would not reach any completion. Also, the continuum is not constructed from the bottom up from some basic elements; it can only be divided from top to bottom, except that this bottom is beyond reach – since it simply does not exist. Hence, "it is plain that every continuum is divisible into [divisibles that are] always divisible" (231b15-16), and, in fact, this statement can be considered a definition of the continuum (232b24-25).

---

[12] Cf. Jaakko Hintikka, *Time and necessity*, Oxford: Clarendon Press 1973, 119-122.

[13] Cf. Bröcker, *op. cit.*, 271; Hintikka, *op. cit.*, 120.

This is true for time, magnitudes, and motion. Although the continuum is only potentially divisible, with this statement Aristotle returns to Anaxagoras' understanding of infinity. Anaxagoras maintained that there is no limit in dividing things, since "of the small there is no smallest" (59B3). Aristotle retained this idea only in the case of continuous things and only as an unrealizable possibility: there is no limit for dividing such things, and the division process is guaranteed to reach no atoms, simply because there are no atoms.

A continuous whole is composed of continuous components which must not be indivisible and which repeat in miniature the whole itself. In this respect, Aristotle's understanding of the continuum is similar to the modern understanding since continuum is made of infinite continua; however, unlike in Aristotle, the modern continuum does have indivisible parts, or points. Aristotle discarded points from the continuum at the cost of a paradox: the continuum can be divided, since "none of the continua is without parts" (233b31-32), but there are no atom-like, ultimate parts composing it.

This division process is also a possibility. Because infinity by definition excludes completion, each division enables another division which has no limit. Therefore, for Aristotle there exists "not a possible infinity but an infinite possibility"[14] and "the way to infinity consists in the infinity of the way [which is the division process]."[15] It is a possibility to go beyond the limits of what has already been done with no chance of ever finishing this process of surpassing newly reached horizons. Trying to reach the end of this process is a never ending quest for completion because of the limitations of nature and the impossibility of breaking from the confines of time. The nature of infinity as given in its definition makes it unrealizable. Its nature thus lies in its nonexistence, its essence in incompletion. Infinity embedded in the concept of continuum has therefore no chance to become actual, to ever have a "separate existence" since it exists only "for knowledge" (γνώσει, *Met.* 1048b14-15). Infinity in any form – number, mathematical magnitude, or "what is outside the heaven" – "never stops [in dividing or magnifying] in thought" (ἐν τῇ νοήσει, *Phys.* 203b22-24). But even this existence in thought is never completed, because, for example, "an infinite series cannot be traversed in thought" (*Anal. post.* 83b6). Regardless of the position in the series, thought can always go to the next position, never seeing the entire sequence.

This, in fact, is rather interesting since thinking itself is infinite (*Phys.* 208a20), and, therefore, it should be able to deal with infinity. However, thinking is infinite in the same sense as time and movement are: they all are in the process of actualizing – and then passing away (208a20). This indicates that Aristotle considered real only that which is happening now. Past and future are in this sense not real. Thought processes and movement are bound by time, they are immersed in time, so that their temporal character renders thoughts unreal, if not happening presently. Time, therefore, becomes implicitly a part of the definition of infinity, and hence actual infinity would be possible in two ways: either by performing anything outside of time in some atemporal zone of the universe, which Aristotle's physics does not allow, or by performing an infinite division in one instant of time, which possibly could be done by Aristotle's God; however, because such an activity does not befit God (his activity consists only in thinking about himself), this would not be a viable solution either.

---

[14] Bröcker, *op. cit.*, 267.

[15] Wolfgang Wieland, *Die aristotelische Physik*, Göttingen: Vandenhoeck & Ruprecht 1992 [1962], 293.

Another source of Aristotle's troubles with continuum is the definition he used: things are called "continuous when the touching limits of each become one and the same" (*Phys*. 227a11-12). But this, strictly speaking, does not define the continuum, but continua. Can an interval AB be considered a continuum? If yes, what can be done with extremities of this interval? After all, they do not touch anything. Hence, a closed interval [A, B] is not a continuum, but an open interval (A, B) with the extremal points excluded is a continuum. This assertion is corroborated by Aristotle's statement that out of all movements, only circular locomotion can be continuous (and eternal) – that is, infinite – so that rectilinear movement is not continuous, at least not in the full sense of the word. The reason is that the continuity of this movement is broken by reaching an extreme, A or B, of an interval and by the need to return to continue the movement (261b31-34).[16] This pronouncement seems, however, to be too strong, since except for points A and B, movement in an open interval (A,B) is continuous, and this continuity can never be broken if a certain condition is imposed on the speed of the moving body, then it is possible. A half of an interval AB is traversed with speed v, a half of the remaining half with speed $v/2$, a half of this half's half with speed $v/4$, etc.; therefore, the half of AB is traversed in time t, a quarter of AB in time t, one eighth in time t, so that infinite time is needed to traverse AB. Because length and speed are continuous magnitudes, the series of such bisections is plausible. This way of moving in an open interval would also render false Aristotle's statement that a finite interval cannot be traversed in infinite time (233a32, 238a17-18).

Also, since rectilinear movement in an interval [A, B] is a movement after all, it can be measured by time, and because some correlation between time and distance can be established, time measuring rectilinear movement would not be considered continuous. But time does not stop when making a U-turn at one of the extremal points, A or B.

The nature of the extremities is also obscure. Are they just sides of interleaving continua? If yes, then denying them the status of points is not justified. If not, then they can be subdivided into continua so that the extremities dissolve in the continua composing them. At one point Aristotle said about any continuum AB that "there is something synonymous between its limits" (234a8-9), i.e., another continuum, CD. If limit A is not identical with limit C, then there is always something intermediate between A and C – even an infinite number of such somethings, which are continua of the same category as the continuum AB. Hence, between any two extremities A and B we can find an interval – even an infinite number of intervals. Aristotle would not say, as it is stated today, that between two points A and B we can always find another point – even an infinite number of them. This statement implies what Aristotle said (an infinite number of intervals between some extremities AB) but not vice versa. In that respect, Aristotle's understanding is limited and rather unintuitive.

Because of Aristotle's approach to continuum, his treatment of time is also counterintuitive. The present or the now (τὸ νῦν) connects the past and future (222a10-11), but it is also "necessarily indivisible" and yet "it is inherent in all time" (233b33-35). First, how can an indivisible now be "inherent in all time" unless being inherent is different from being a part? It must be different, since time as a continuum must have no indivisibles: "there can be no primary [part of time]" (239a20). Also, "nothing is in motion in a now" (234a24); thus, measuring velocity at a certain point is impossible. But if motion expresses time and time expresses motion, why should not the now and a point-motion (and point-rest) be treated interchangeably? Aristotle's statements are defended on the grounds that a correlation of the nows, points of space, and point-motions is a correlation of limits, "not assigning occupancy of some indivisible part of space for some indivisible part of time,"[17] which is not convincing since the correlation of points between two continua should imply "assigning occupancy" (although the latter phrase is rather cryptic). Also, time has a special status since the past is gone and the future does not exist yet, thus, time, in a way, does not exist either; only the now does, but the now, being a point is not even part of time.[18] Paradoxically, the now, the present instant, which is the most real, most palpable time in human experience, is not considered to be part of time in Aristotle's physics. Real time is not time and time is what does not exist. If God could see time from his extratemporal perspective, time would not be a collection of nows anyway. As a continuum, it is not composed of temporal points. Another paradoxical consequence of this view is that a change has a beginning and end but no point at which it begins or ends: an object is neither at rest nor in motion in a now, so no such point can be used for determining a change in the object.[19]

This curious status of points (and the nows) indicates that if (as surmised above) God wanted to divide a continuum indefinitely, he would destroy its structure right after the first division. A continuum can be thought of, but not actually divided; we can define what it can be, but when trying to actualize this possibility, we destroy the continuum.[20] It is like trying to read an old scroll that is so brittle that it crumbles when touched; if we could unfold it, we would see the script and read it, but the actual process of unfolding destroys it.

---

[16] The problem is discussed in Adam Drozdek, Continuous motion in *Physics* viii 8, *Acta Antiqua Academiae Scientiarum Hungaricae* 43 (2003), 59-65.

[17] Edel, *op. cit.*, 80.

[18] Cf. Ursula Coope, *Time for Aristotle*, Oxford: Clarendon Press 2005, 12.

[19] Sebastian Wolf, *Das potentiell Unendliche: die aristotelische Konzeption und ihre modernen Derivate*, Frankfurt/M.: Peter Lang 1983, 118-119, 123. Aristotle himself admitted that "there is no beginning of a change" (*Phys.* 236a14).

[20] Wieland, *op. cit.*, 302. It is stated that "we come to recognize that the impossibility of an actual infinite division of matter is not ultimately a fact about matter, but a fact about ourselves: we are the dividers, and we are capable neither of carrying out nor of witnessing an actual infinite," Jonathan Lear, *Aristotle: the desire to understand*, Cambridge: Cambridge University Press 1988, 67. Actually, it is primarily a fact about matter and only incidentally about ourselves.

11.5 CAUSALITY

The problem of infinity becomes acute in a discussion of causal chains. "Everything that is in motion must be moved by something" (*Phys.* 241b24), says Aristotle, so that there exists a series of objects, one causing the movement of another. However, "there must be some first mover and [the series] cannot go on to infinity" (242a19-20). Does it contradict the statement that movement is eternal?[21] If there is a UM which stands at the beginning of this series of objects, then the movement has a definite temporal beginning. However, the action of the UM can be considered not as constituting the beginning of the series of movements, but as parallel to this series. There is no definite point of time when the UM first became the initiating cause; it has always been such a cause, from eternity. Aristotle's razor, on the one hand, prohibits the infinity of the causal chain; on the other hand, when combined with eternal movement, it apparently necessitates an infinite influence of the UM on the world. The razor then, instead of reducing infinity, spawns it.

It is important to note that an infinite causal chain is impossible only because of Aristotle's razor since, theoretically, such a chain with no beginning (and no end) could exist – principles of Aristotle's physics do not preclude such a possibility. A theory combining this physics with an infinite causal chain could be considered simpler since it would not require theological underpinnings (cf. Theophrastus). However, the prohibition to extend the causal series into the past is a primal source of many problems, one being the type of causality exercised by the UM.

Aristotle appears to have given two ways in which the UM acts on things: the causal-mechanical way specifies acting through touching, and the teleological way concerns an immanent desire of things for the mover. Hence, "the prime mover – [understood] not as 'that for the sake of which' [= a final cause], but a source of motion [= a moving cause] – is always together with that which is moved" (*Phys.* 243a3-4) and such physical contact is observed in all cases (242b25-27, also 202a7-9, *De gen. anim.* 734a4). Therefore, "such a principle seems to suggest that realities of the same nature are under consideration, realities being in contact."[22] After all, Aristotle locates the UM on the circumference of the universe to move the first, astral sphere (*Phys.* 267b6), but this does not square well with the fact that the UM has no magnitude and that it is an eternal immovable substance. Such substance, unlike sensory substance, is immaterial and hence occupies no space (and there is no space outside the first heaven); therefore it has no location and cannot touch anything, thereby being an efficient cause. But there is another, teleological meaning of the UM as being an inceptor of movement: the UM is an object of desire of the universe and "it moves as an object of love" (*Met.* 1072b3). In this way, the UM is only the final cause and its causality through contact should be treated at best metaphorically: it is possible, when imparting motion, to touch without itself being touched (*De gen. et corr.* 323a31-32); thus moral contact can be meant.[23]

11.6 ETERNITY

Why require eternal existence of the UM or of the universe? Because the nature of things is such: "in the order of nature, of definition, and of time, alike the complete is prior to the incomplete and the imperishable to the perishable" (*Phys.* 264a22-23) and also "all causes are by necessity eternal" (*Met.* 1026a16-17). This non-physical reason indicates that circular movement precedes rectilinear movement, hence eternal movement precedes perishable movement, eternal time is better than time with an end. This is a remarkable suspension of Aristotle's razor: infinity limited as much as possible to the temporal sphere takes precedence over the finite, and the latter has to be explained in terms of the infinite. It is said "as much as possible," since the spatial aspect of infinity is not ruled out altogether. Rectilinear infinity is cut off by Aristotle's razor, but a body moving indefinitely on a circumference covers a path which is infinite,[24] thus spatial infinity expressed in a tamed form, in the form of a circle, is acceptable, since it can be confined to the limits of the universe. The universe itself, however, has a finite dimension, and this appears to be a prime application of Aristotle's razor.

As mentioned, the UM is an object of desire of the universe and thereby it becomes the final cause. As an object of love, the UM is the goal which everything attempts to approach. The UM itself, however, is detached from the world; it is not just disinterested in it, but does not even know about its existence. Being loved by the universe, it does not love anything. And yet, since the UM is perfect, the universe desires to match its perfection in the highest degree possible, also with respect to the kind of movement. Therefore, because circular movement is pronounced to be most perfect, the first heaven moves circularly, which would explain the question: "what is a relationship between knowing and desiring ... and the fact of rotation?",[25] "what intelligible connection ... between circular movement and love"?[26] The UM and its eternity is the highest object of desire of the first heaven and the first heaven actualizes the divine eternity in the only way it can, namely as circular movement. And since this is an infinite movement (both timewise and spacewise, or rather pathwise), infinity stands at the beginning and at the end of everything.[27]

---

[21] Cf. Klaus Oehler, *Der Unbewegte Beweger des Aristoteles*, Frankfurt/M.: Klostermann 1984, 53.

[22] Michel P. Lerner, *La notion de finalité chez Aristote*, Paris: PUF 1969, 115.

[23] René Mugnier, *La théorie du premier moteur et l'évolution de la pansée aristotélicienne*, Paris: Vrin 1930, 113.

[24] This may not be quite in agreement with Aristotle, since such a path would extend infinitely only into the past and "infinity in one direction is neither infinite nor finite" (*De caelo* 283a11). Admittedly, there is no substantiation of this highly artificial claim, but a charitable explanation may indicate that this is probably because one-directional infinity has one limit, which would lead to a nontrivial problem of apparently having different infinities: two semi-lines extending indefinitely in two directions would be twice as long as the entire line. This did not cause a problem for divisions, since by definition a part of an interval was infinitely divisible as the entire interval, but it is not equally obvious whether the length of a line extending in one direction is the same as the length of a line extending in both directions (assuming that such a length could be measured, which by definition of magnitude is ruled out in Aristotle's physics).

Similarly, eternal existence as such is a final cause of animate objects. A plant and an animal reproduce "in order that, as far as its nature allows, it may partake in the eternal and the divine. This is the goal towards which all things strive" (*De anima* 415a30-b2) where 'divine' seems to be a repetition of 'eternal' and is not restricted to the UM. Supralunar spheres are in that sense also divine, since they are imperishable, but they move, whereas the UM does not. But this statement concerning the meaning of reproduction is almost a quotation from Plato, who said that "through generation mortal things participate in perpetuity and in immortality" (*Symp.* 206e-207a) since "mortal nature seeks as far as it can to perpetuate itself and to be immortal" (207d). This is a natural desire of men as well; therefore, they should marry and procreate (*Laws* 773e, also 721bc).

The whole of nature is directed toward this supreme mover, and the order in the world is guaranteed by this directedness. All the world's potentiality is founded upon God's actuality brought to the highest degree. Therefore, "nature does its best to imitate mind," which is God.[28]

The UM is unmoved and eternal, and its eternal character guarantees that the world is eternal and that the universe is self-contained and permanent (*Phys.* 259b21-28). Because "the perfect is naturally prior to the imperfect" (*De caelo* 269a19), the UM is the most perfect of all things, "a perfect being" (*Met.* 1073a1), and as necessary, it is good (1072b10-11), and its characteristics also are most perfect and desirable. Therefore, as in Anaximander and in all philosophers after him, the infinite – at least in the form of the eternity of the UM – stands at the beginning of all things, causing their existence and movement. The UM is of infinite duration and, to some extent, it imparts temporal infinity to other things; therefore, the finite and the temporal can be understood only in the light of the infinite, since the infinite precedes the finite. The infinite cannot be banned from the universe, since it enables its very existence. This is what Aristotle proved, and yet he made his best efforts to limit the impact of infinity upon the world and narrow its presence in nature.

In spite of the exalted status ascribed to the UM, Aristotle said in another context, "nature flies from the infinite, for the infinite is unending or imperfect, and nature ever seeks amends" (*De gen. anim.* 715b15-16). Aristotle could defend his position by saying that what he meant was an actual infinity. Is the UM, therefore, only potentially infinite? First, we have to remember that Aristotle was very cautious in that he did not call the UM infinite. This mover is simple, thus indivisible, thus not infinite with respect to spatial divisibility; moreover, it has no magnitude because it has no body, because "the primary essence [= the UM] has no matter" (*Met.* 1074a35-36); it is neither infinite nor finite and thus, there is nothing that can be predicated as either infinite or finite. Besides, we cannot say about any object that it has a certain property or it does not. Does the atom (in today's sense) have a color? Color is not a property of atoms, hence asking this question in regard to atoms is of little sense.

But the UM is undeniably eternal so that it can constantly impart movement to celestial spheres which are also eternal, thus at least infinity in respect to duration can be found in it. In what sense? It appears that the main reason for which Aristotle needed the UM was to account for the eternity of movement and time as its measure (hence, to a large extent, time and motion can be treated interchangeably).[29] Since this time has no beginning, and so does not motion, the UM is also characterized by infinity, or rather, the infinite duration of the UM guarantees temporal infinity of the world. But Aristotle would consider this only a potential infinity since time does not exist in its totality.[30] However, for Aristotle, potential is akin to non-existent; actual, however, is severely limited in scope. But a change of terminology could restore the status of time: let us call what happened in the past and what is happening now real, so that time can be called real, although potential. It is actual only now, and the current events become only potential in the future, but they are nevertheless real just as the time in which they occurred.

Extratemporality of the UM is assured in Aristotle's system since for him it was evident that there is "no place or void or time outside the heaven" (*De caelo* 279a12-13). Thus, the UM is timeless, "since what does not move nor rest, is not at all affected by time."[31] This can be explained on terminological grounds that time does not age whatever is outside the heaven (279a19), that this sphere is not

---

[25] Lerner, *op. cit.*, 135.

[26] André Bremond, *Le dilemme aristotélicien*, Paris: Beauchesne 1933 [reprint, New York: Garland 1987], 108.

[27] These considerations rely on the assumption that circular movement really is the most perfect of movements, which is what it is – an assumption of no other provenance than Plato's doctrine. However, it is superfluous and Aristotle's physics would not be harmed by avoiding it, Enrico Berti, La suprématie du movement local selon Aristote: ses conséquences et ses aporie, in J. Wiesner (ed.), *Aristoteles: Werk und Wirkung*, v. 1, Berlin: de Gruyter 1985, 137, 145. Also, the kind of movement of the body affected by the UM depends on "the nature and the perfection" of this body (p. 147). But it is not true that such a body (here: the first heaven) "moves by itself": this body requires the UM's action, but the body moves the most perfect way it is able to, and this way is independent of the UM.

[28] Lear, *op. cit.*, 307.

[29] Time is measured by movement and vice versa (*Phys.* 220b23-24). Time is not motion but "number of motion" (221b11).

[30] For a comprehensive discussion of the problem of time in Aristotle see Jacques M. Dubois, *Le temps et l'instant selon Aristote*, Paris: Desclée de Brouwer 1967. A distressed author concluded his book on this problem with the statement that time in Aristotle "escapes characterization with any well-defined category. What Aristotle really gives us about the essence of time is insufficiently unequivocal to secure any clear notion and too formless to become sharper," Paul F. Conen, *Die Zeittheorie des Aristoteles*, München: Beck 1964, 172.

[31] Gustav Kafka, *Aristoteles*, München: Reinhardt 1922, 55; the UM does not move, but does it not really rest?

"in time,"[32] but this explanation can be considered a way of allotting the extracelestial domain an even loftier status: the infinity of time and movement is guaranteed by the effects of action of this domain, thus its status is even higher than the temporal infinity, which can be nothing else than infinity of the second order.

11.7 THE INFINITE AND THE FINITE

Aristotle built a grand system, but he built it as a physicist, as a scientist trusting more his senses and relying more on observation than on hypotheses. It is very symbolic that the name of metaphysics implies coming after the physics, almost being an afterthought, an epiphenomenon of physics. This marks a very strong breach between Plato and Aristotle. For Plato, the truth was to be found in the extraperceptual domain, in the world of ideas whose faint reflection can be found in the world around us. In particular, Plato had no problem with infinity in its spatial and temporal aspects. Aristotle, however, started with a common sense observation that our perceptual abilities are limited, hence infinity cannot be a part of our cognitive apparatus. This was certainly the reason for defining infinity as something always incomplete. We can wonder, therefore, how he could maintain that facts, i.e., probably observations, are to prove the eternity of circular movement (*Met.* 1072a21-22)? Strictly speaking, eternity (or existence) of circular movement of the heaven can in no way be observed.[33]

Human cognitive limitations were the reason for relying on Aristotle's razor. These limitations are expressed in many places as the fact that "the infinite as infinite is unknowable" (*Phys.* 187b7); that "objects so far as they are an infinite manifold are unintelligible" (*Anal. post.* 86a6); that no systematic knowledge of infinitely various cases is possible since "how can one think about things that are infinite" (*Met.* 994b22-23), "how is it possible to get knowledge of the infinite [number of] individuals" (999a27-28)? These rhetorical questions have one obvious answer and this answer has been given in the form of Aristotle's razor which expresses the *horror infiniti*. This fear of infinity is not infrequently ascribed to all Greek thinkers, but it was only Aristotle who succumbed to it, probably for two reasons: his displeasure with the highly abstract and "unearthly" system of Plato, and Zeno's paradoxes.

The difference of his approach in comparison to Plato's is also symbolically summarized in the name of the peripatetic school, which would indicate that Aristotle was firmly treading the earth, gazing at the stars from the earthly perspective. Plato, on the other hand, was soaring high above the earth and looking at the earth from the perspective of the beyond. The material world was an instantiation of the infinity of the world of ideas; for Aristotle infinity was an unachievable goal worth considering only to be rejected. It is potentiality in the weakest sense, admissible only in this potential sense, since otherwise paradoxes of physics and

---

[32] Cf. a discussion in Richard Sorabji, *Time, creation and the continuum*, London: Buckworth 1983, pp. 126-127.

[33] Cf. Berti, *op. cit.*, 125.

astronomy would be unbearable, paradoxes of Zeno being examples of this danger.

The problem Aristotle had, as indicated, was an apparent impossibility of handling infinity and of gaining any reliable and systematic knowledge in the face of infinity. However, an example of Peano's arithmetic should indicate that it is not an impossible endeavor. This arithmetic is presented as using only five axioms, but one of them is a schema of axioms; thus there are an infinite number of axioms. Hence, infinity was captured in five statements whereby theorem proving is possible. Also, the laws of nature can refer to an infinite number of instances, and detecting these laws does not necessitate knowing all instances. Infinity can be used by finite beings – and they use it, since without it no knowledge would be possible. Aristotle is a good example of this: his razor was not powerful enough to expunge all infinity from his own system, and although in a convoluted form, its temporal aspect was retained. He attempted to substantiate existence of this infinity by observation, but it could not have been done; such infinity must be assumed. Thus we can agree that "the thesis attempting to base an infinite process on a sensory intuition is radically false ... [since, in particular, experience] does not show us a possibility of indefinite generating of numbers."[34] And that is what Aristotle attempted to show by his heavy reliance on perception.

Aristotle's example should indicate that infinity cannot be supported by the finite. It is the infinite which should be used as a point of departure, as a datum which enables us to gain any knowledge. Otherwise, the system either becomes very restrictive and artificial, or downright contradictory. The assumption of the infinite leads to some knowledge, not its rejection. We can therefore agree with Aristotle's statement that the problem of infinity "has practically always been the source of the differences" among philosophers and scientists (*De caelo* 271b6-7). For Aristotle it certainly was.

---

[34] Francis Warrain, *Examen philosophique du transfini*, Paris: Hermann 1935, 13-14.

12 THE OLD ACADEMY

12.1 THE UNWRITTEN DOCTRINES

In *Letter* vii, Plato stated that there is a sufficient knowledge of "many … and the most important" points that cannot be communicated in words; this knowledge can be gained by the pupil from his teacher through constant verbal communication that leads to illumination, to igniting the spark of understanding in the soul of the pupil (341b-d). There is thus, presumably, part of Plato's philosophy – the most important part, in fact – which cannot be found in his writings, or at least it is there in an imperfect form. Aristotle mentions Plato's unwritten doctrines (ἄγραφα δόγματα, *Phys.* 209b15), and he and then his commentators give some information about these doctrines. According to Aristotle, the ideas were not the ultimate in Plato's system (*Met.* 987b14-988a17). Multiple material entities participated in one idea, but the unity of the infinite world of ideas was assured by two principles, the one and the dyad, the principle of unity and the principle of plurality: "the forms/ideas are the [formal] cause of other things, and the one [is the formal cause] of the forms" (988a10-11). The material cause of "other things," of the material things shaped in accordance to the ideas, and of the ideas themselves that are shaped by the one, is the dyad also called the great and the small (a11-14) and the indefinite dyad (ἀόριστος δυάς, Alexander, *ap.* Simplicius, *In Phys.* 151.7).

With these two principles, Plato's system reaches another level of generality and unity where just two principles are used to account for the unity of all that exists and, at the same time, its plurality. The name for the second principle, the great and the small (or just the great and small) calls to mind Plato's analysis of the infinite offered in the *Philebus*. Chapter 10 claimed that the *apeiron* of this dialogue consisted of continua and ordering relations, i.e., of pairs of the form (quality Q, $<_Q$). One such relation was the greater and the lesser. By generalizing this kind of relation for each continuum, the entire domain of the *apeiron* could be called the great and small to stress the fact of infinite expansion of each continuum into the small and into the great. And, in fact, it is stated about the unwritten doctrines that they propose the construction of infinity out of the great and small (Aristotle, *Met.* 987b26) or out of the indefinite dyad (Theophrastus, *Met.* 6a27-b1) or simply that the indefinite dyad is infinite (Simplicius, *In Phys.* 453.26-27) and of the nature of the infinite (455.9-10), and it is indefinite because it is infinite (Alexander, *In Met.* 56.20). The infinite dyad is a double infinity, increasing and also decreasing, an infinite nature that is called the great and the small since this nature proceeds toward the great and also toward the small (Porphyry, *ap.* Simplicius, *In Phys.* 454.5-9), i.e., the dyad is an acknowledgment that "it is possible to be greater and smaller ad infinitum," (Hermodorus, *idem* 248.6-7). In this way, infinity acquires even more prominent position in the unwritten doctrines than in

Plato's written works. Infinity is explicitly the foundation of the universe, a principle that requires limitedness and determinedness of the one to render the existing beings. The dyad is thus an elaboration of the domain of the *apeiron* discussed in the *Philebus*. The one of the unwritten doctrines would be a counterpart of the *peras* of the *Philebus* which was responsible for unity, harmony and order imposed onto the domain of the *apeiron*. In the *Philebus*, Plato was speaking about the mixture of the *apeiron* and the *peras*; in the unwritten doctrines, the one is a formal principle, the dyad is a material principle, but the result is the same: the ordered and unified universe due to universal principles. Therefore, the doctrine of the two primary principles of the unwritten doctrines can be considered an elaboration of the proposal set in the *Philebus*.[1]

The unwritten doctrines seem to become written, although in distinctly modified form, by Plato's successors.

12.2 SPEUSIPPUS

According to Aristotle, Plato distinguished three eternal substances – the forms, the mathematicals, and the sensibles. This ontology was modified by Speusippus, the successor of Plato as the head of the Academy, who "made still more kinds of substance, starting from the one, and making principles for each kind of substance, one for numbers, another for spatial magnitudes, and then another for the soul" (*Met.* 1028b21-24 = F29a Tarán). Importantly, among Speusippus' principles there are no ideas/forms, which marks a major departure from Plato's ontology. Aristotle very likely refers to Speusippus when he mentions those "who make the mathematicals [exist] apart from the sensibles seeing the difficulty and fictitiousness about the forms, [and who] abandoned ideal number and posited mathematical" (*Met.* 1086a2-5 = F35; cf. F34).

Based upon a general principle that "generation is from contraries," Speusippus apparently claimed that in the generation of numbers, two contrary principles are involved, the one and the plurality (1091b30-35 = F45a), which indicates that for each substance there should also be two contrary principles from which the substance emerges: for example, the point and "other matter like plurality, but not [identical to] plurality" are principles of magnitude (1085a32-34 = F51).

Because sensible substances are generally assumed to exist (1069a31 = F31), it is very likely that Speusippus distinguishes at least four substances: numbers, spatial magnitudes, souls, and sensibles. To learn more about Speusippus' ontology, we may refer to ch. 4 of Iamblichus' *De communi mathematica scientia* that is believed to present Speusippus' ideas.[2]

[1] A close connection between the unwritten doctrines and the *Philebus* has already been obvious to Porphyry who discussed the dialogue in the light of these doctrines (Simplicius, *In Phys.* 453.30-454.19). See also Giovanni Reale, *Toward a new interpretation of Plato*, Washington: The Catholic University of America Press 1997, chs. 14, 17.

[2] Philip Merlan, *From Platonism to Neoplatonism*, The Hague: Nijhoff 1960 [1954], 100; John Dillon, Speusippus in Iamblichus, *Phronesis* 29 (1984), 325-332.

This chapter suggests the existence of five kinds (γένη), but only two are discussed: the first kind, numbers, and the second kind, which includes lines, solids, and flat areas. The fourth kind and the fifth kind are only mentioned as the last ones ("at the extreme" of the hierarchy, 18.9), and the third kind is not mentioned at all.[3]

Different kinds of being are ordered into different hierarchical levels. However, hierarchical dependence is of ontological, not temporal order since Speusippean substances are eternal: in F29a, Speusippus' substances are put side by side with Plato's, which are eternal, so Speusippus' substances should also be considered eternal. Therefore, for example, numbers did not exist before magnitudes, nor vice versa. It is rather that if numbers had ceased to exist, magnitudes and any other substances would have ceased to exist as well, but in the event of the annihilation of magnitudes the existence of numbers would not have been undermined. Speusippus uses a generational language, but the language should be taken to indicate the eternal causal dependence between different levels of being.[4] In particular, the eternity of geometrical objects is explicitly stated. Speusippus said that in the theoretical science, the objects of theoretical sciences should be called theorems, not problems, because these sciences are about eternal things; therefore, geometrical proofs and geometrical constructions are not creating new geometricals, but are ways of understanding them, "taking beings that always are as though they were [in the process of] coming to be" (F72.10-11). It may be that this procedure is necessary because we are finite and thus limited beings. The view of eternity of eternal beings can be gained through finite means only with constant awareness that these means are, as it were, necessary evils that allow us to gain an insight into the eternal and the infinite. They would be a means of making the truth of mathematical statements the "delight to the souls" (F36.29), possibly in the spirit of the *Meno*.

First, numbers are generated and both highest and first principles, the one and the plurality, are participating in this act, plurality being "like some moist and easily molded matter" (*De communis* 15.5-6, 11, 13). This is similar to the unwritten doctrines, in which the one produced numbers from the dyad as from a mold (ἐκμαγεῖον, Aristotle, *Met.* 987b33-988a1). The one thus molds the plurality, giving rise to the first level of being, numbers; or, the plurality stamps out the form of the numbers which is thereby indefinite in plurality but definite in form since the one does the molding (*De communis* 16.15-18). The process is not only indefinite but infinitistic since an infinity of numbers are generated in the process, each number being well rounded, well-formed since it bears a stamp of the one that provides for each number its form. There is thus no separate world of forms (ideas); the forms are provided by the one, or, more likely, by God-*nous* who is different from the one (F58). Infinity of numbers very likely requires infinite matter, i.e., the easily moldable matter of the plurality should be infinite.[5] In fact, that much is stated explicitly: since "someone posits one infinite matter and receptacle for all [things]" then the one by itself would generate everything alike (*De communis* 16.18-22).[6] That is, the one is molding plurality which is a preexisting infinite matter. This infinite numerical matter indicates that an infinity of numbers exists, which would correspond to an infinity of Platonic ideas.

The second level of being, the second kind of substance, consists of geometricals which are created with two contraries, the point which is a counterpart of the one used for numbers, and a receptacle or matter proper for geometricals (17.13-19). Whence these two principles? Apparently they both exist from eternity; however, because of ontological dependence, their existence depends on the one and on malleable numerical matter; that is, the point and geometrical matter are reflections, lesser images of the one and numerical matter. It is stated that matter is of lesser quality than numerical matter since geometricals are tougher and denser than numbers due to their material principle (17.20-22). And again, geometrical matter should be expected to be infinite since there is an infinity of geometrical figures.

The origin of numbers and geometricals seems to set the pattern for other kinds of being as well, namely a pair of contraries proper for each kind, a formal principle and a material principle, corresponding to the one and numerical matter.[7] Each kind appears to be a lesser reflection of the higher kind, and because of their lesser status and quality, each kind and its principles ontologically depend on higher kinds and their principles. Therefore, a supposition may be ventured that the third kind of being is that of the soul since it is stated that the soul emerges from number and magnitude, not being identical to them (F30, F59), possibly with the counterpart of Plato's world soul being the formal principle and some psychic matter being the material principle. A still lower kind of being would be the level of the physical world and physical, palpable matter. Physical reality seems to occupy the two last levels of being, the fourth very likely being the world of imperishable physical entities (the world as a whole, the stars), and the fifth, the world of perishable physical entities.[8]

These, however, are just suppositions since details pertaining to the last three kinds of being are not provided in *De communis* iv. It is also unclear what exactly the status of the one is. Sometimes the one is mentioned as directly involved in the generation of numbers so that the one and the plurality are "the first and highest principles" posited for numbers (15.6-7, 16-17); at another time, however, the one is distinguished from "the monad according/corresponding to the one" which is used to generate numbers (17.15). Therefore, it is sometimes conjectured that there are five pairs of contraries (monad-plurality, point-geometrical matter, etc.) with the one constituting a separate entity,[9] or that the one is the member of the first contrary pair and the monad is just another name for the one since some deliberate terminological laxity is explicitly expressed for matter ("the first receptacle, or magnitude, or whatever it should be called," 16.15-16).[10] It seems that the latter position is more likely. In this way, there are five contraries, thus ten principles, because the number ten, the *decad*, has a special significance in Speusippus' numerology as "the most natural and most perfective of beings, a sort of productive form for the cosmic accomplishments," "as an all-perfect paradigm" (F28.10-13). Also, special status appears to be ascribed in this ontology to God-*nous*, therefore, it would seem to be too much to accept also the one between God and the ten principles. Moreover, would not symmetry also require some special primal matter to be assumed to exist of which all five matters in the five contraries would be some kinds of reflection?

---

[3] Concerning the problem of identification of the substances and principles, see H.A.S. Tarrant, Speusippus' ontological classification, *Phronesis* 19 (1974), 131, 134.

[4] R.M. Dancy, *Two studies in the Early Academy*, Albany: State University of New York Press 1991, 85.

[5] This is not necessarily true: with the bisection argument it can be shown that infinity of entities can be generated from a finite amount of infinitely divisible substance.

[6] There is an echo here of the *Parmenides* 158d where Plato mentions the one and the infinite rest upon which the one imparts limitedness.

[7] Cf. Merlan, *op. cit.*, 100.

[8] A supposition is also made that the fourth level consists of animate beings, the fifth, that of inanimate beings, Merlan, *op. cit.*, 116; John Dillon, *The heirs of Plato*, Oxford: Clarendon Press 2003, 55.

The existence of the correspondence between the contraries proper for each kind of being may mark another departure from Plato's ontology. The numerical matter is explicitly pronounced to be infinite and the infinity of geometricals very likely requires infinite geometrical matter. Is the infinite amount of matter also carried over to psychic matter and physical matter? If so, the world very likely would be infinite thereby breaking the finite confinements imposed upon it by Plato and Aristotle and returning to an infinite universe as envisioned by most Presocratics. Also, this world would be populated with an infinite number of souls. It is possible that only the finite world would be created, and thus unused matter would constitute some cosmic waste; however, this waste would be infinite and thus it is very unlikely that it would be allowed in the Speusippean universe. Therefore, Speusippus may have assumed that the psychic matter and physical matter are finite, whereby the finite size of the universe and the finite number of souls would be preserved. In this way, because of the decreasing level of perfection in the hierarchy of beings, infinity would not be present at all levels and, in fact, an assumption can be ventured that the ontological hierarchy proposed by Speusippus is based on the level to which each kind of being and its two contrary principles are saturated with infinity. The first two levels, of numbers and of geometricals, have infinite matter and thus an infinite number of entities: an infinity of numbers and an infinity of geometricals. However, the world of geometricals is not as uniform as the world of numbers since among geometricals there are points, line intervals, and two- and three-dimensional figures. Also, geometricals are somewhat more palpable than numbers and thus geometrical matter is tougher and denser than numerical matter, resulting in less sublime entities – i.e., geometricals – than numbers.

The third level of being, the level of the soul, is saturated with infinity only partially: the psychic matter is likely to be finite and thus there is only a finite number of souls. However, the souls are not only immortal (F55), but possibly also ungenerated when we take into account Speusippus' metaphorical interpretation of the *Timaeus* as written "for didactic reasons," "not implying that the universe really had a beginning" (F61ab), thereby denying that generation is a temporal process and believing that the world was ungenerated and so was the soul[11] as also suggested at one point by Plato in the *Phaedo*.

The last two kinds of being probably both have a finite material principle and thus are populated with a finite number of entities that exist eternally (level four) and those whose existence has only a finite duration and thus are perishable (level five). Infinity, thus, would mean perfection and finitude – imperfection. This hierarchy of decreasing perfection would be an ontological explanation for why the amount of physical matter is finite and, consequently, why the world – as posited by Plato – is finite, as well as why physical level does not guarantee eternal existence the way the level of the soul and upper levels do.

In this way, Speusippus would make infinity an important element of his ontology, basing it on the level of saturation of a particular level of being with infinity. Infinity, which has such a prominent position in Plato's system, but many a time only implicitly, would become the foundational aspect of Speusippean ontology.

12.3 XENOCRATES

Xenocrates, the head of the Academy after Plato and Speusippus, to a large extent restored Plato's orthodoxy primarily by restoring the ideas (forms). In his view, a form is "a paradigmatic cause of always naturally constituted things … a separate and divine cause" (fr. 30 Heinze).

According to Sextus, Xenocrates stated that "there are three substances/beings: the sensible, the intelligible, and synthesized [which is also] the object of opinion (δοξαστή). The sensible [substance] is inside the heaven, the intelligible is [the substance] of everything outside the heaven, *doxaste* and synthesized [is the substance] of the heaven itself" (SE 7.147 = fr. 5). That is, Xenocrates presents a tripartite ontological division of the universe – above heaven, heaven, below heaven – which corresponds to Plato's distinction between the immutable world of ideas, an intermediate mixture, and the corporeal realm (*Tim.* 35a),[12] and to a tripartite epistemological division – intelligible knowledge, opinion, sensory perception.[13]

---

[9] Merlan, *op. cit.*, 106, 130-131; Jens Halfwassen, Speusipp und die Unendlichkeit des Einen: Ein neues Speusipp-Testimonium bei Proklos und seine Bedeutung, *Archiv für Geschichte der Philosophie* 74 (1992), 55, 63 note 79; Dillon, *op. cit.*, 44-45. The one and the plurality are also considered to constitute the first level, with monad and plurality being on the second level (then, the level of geometricals, then the soul level, and finally the level of sensibles), Alain Metry, *Speusippos: Zahl – Erkenntnis – Sein*, Bern: Paul Haupt 2002, 117, 126, 155.

[10] Leonardo Tarán, *Speusippus of Athens*, Leiden: Brill 1981, 36, 100 note 444, 313. And thus "we should not attach too much importance to the difference between the terms 'monad' and 'One'," as remarked by Merlan, *op. cit.*, 107. Other interpretations also exists: there is only the one manifesting itself in different guises and five different material principles, A.S. Tarrant, *op. cit.*, 135-137; Dancy, *op. cit.*, 108; there are different formal principles with only one material principle, the plurality, manifesting itself in different guises, Dillon, *op. cit.*, 45, 55.

[11] Tarán, *op. cit.*, 47.

Another report of Xenocrates' ontology is provided by Theophrastus, according to whom Xenocrates "somehow assigns everything its place around/in the cosmos,[14] alike sensibles, intelligibles, mathematicals, and divine things as well" (*Met.* 6b7-9 = fr. 26). The four domains listed here can be reconciled with the tripartite division indicated by Sextus when intelligibles and mathematicals are occupying the area outside the heavens and the divine things are identified with the heavenly realm, in particular, with the star-gods.[15] Since ideas are certainly counted among the intelligibles, this points to giving an equal status to ideas and mathematicals, that is, in particular, numbers are ideas, which can be confirmed by Aristotle's mention of "those who say that forms and numbers have the same nature and the rest depend on them, namely lines, surfaces and so on until it comes to the substance of the heavens and to sensibles"[16] and Asclepius indicated that the fragment refers to Xenocrates (*In Met.* 379.17 = fr. 34).

Arguably, the most important part of Theophrastus' statement about Xenocrates is the emphasis placed on the fact that Xenocrates' explanation of the tripartite universe is comprehensive in that everything is derived from the one and the indefinite dyad, whereas others, for instance, derive from them numbers, surfaces, and bodies (or solids) leaving everything else unexplained (6a24-28, b6-7). Some details about the nature of these two highest principles are seemingly provided by Aetius' testimony which begins with the statement that Xenocrates considers "the monad and the dyad to be gods; the former as a male [principle] has the role of a father that rules in heaven, and calls it Zeus, the odd [number], and *nous*,[17] that is for him the first god; the other [the dyad] as a female [principle] in the manner of the mother of gods rules over the region below the heaven and she is for him the soul of the all" (Aetius 1.7.30 = fr. 15). The monad is the highest God, like Zeus among the Olympian gods, God the father, the ruler of the universe, the active principle that is responsible for what the universe is. It is clear that the monad is Plato's Demiurge who molded the gods and the cosmos from preexisting matter. Fr. 15 also seems to indicate that the dyad is the world soul, the soul of the all as also called by Plato (*Tim.* 41d), the mother of the gods, whereby the world soul has a more active role in Xenocrates' universe than in Plato's. However, the dyad-world soul described here can hardly be the second highest principle since the world soul is not the primal entity in the Xenocratean universe. According to the report Plutarch made when discussing Plato's account of the formation of the world soul by the Demiurge, Xenocrates and his followers believed that only "the generation of the number is signified by the mixture of the indivisible and divisible substance; for the one is indivisible, the plurality is divisible and from them is made the number by the one bounding the plurality and by imposing limit upon unlimitedness (ἀπειρία) which they also call indefinite dyad." Then the one and the dyad transform the number into the soul by adding the same (the principle of rest) and the different (the principle of change) (*De an. procr.* 1012d-1013a = fr. 68). The description refers to the complicated process of creating the world soul by the Demiurge from a mixture of three mixtures: divisible being and indivisible being; the divisible same and the indivisible same; the divisible different and the indivisible different (*Tim.* 35ab). The one, the highest immutable God, exercises his power on the primal matter, the divisible plurality, by stamping out numbers and – because they are of the same nature as numbers – ideas from it, thereby mixing his definiteness and orderliness with the indefiniteness of the plurality which is the primal indefinite dyad. Adding mobility to the mixture leads to the emergence of the soul. Because of this kinship relation of the number and the soul, the latter is called a self-moving number (frs. 60-65).

Some elements of Aetius' and Plutarch's reports appear in late and apparently somewhat confused testimony of Favonius Eulogius who, in a chapter on the monad, reports that "Xenocrates thought that number is mind and god: for there is nothing except what is subject to it. But that itself, which is only one and single, is before everything, in everything, and after everything. Whatever amount you would want to count, it starts with one, it is increased by one, and it stops at one, and although all [amounts] that can include it, perish, it will remain immutable" (*De Somnio Scipioni* 5.6-7 = fr. 16). The mind (*animus* = νοῦς) and god clearly correspond to the one of fr. 15; however, equating it with number is contradicted by Plutarch's testimony. The justification ("whatever amount ...") is perhaps Favonius', but the testimony clearly indicates the elevated position of the one, the first primal principle. The dyad is not mentioned, probably because the fragment appears in the chapter on the monad, but the derived character of numbers from the immutable one (*unum* = ἕν) is strongly emphasized.

According to Xenocrates, there appear to be two highest principles: the monad (the one, the Demiurge) and the indefinite dyad, both constituting the origin of everything else. Fr. 15 is theological in nature and has a list of different gods, the two highest gods being Zeus, ruling in heaven, and the world soul, ruling below heaven. Because of the distinguished positions of these two gods, the world soul can also be called the dyad. Therefore, we have the ontological pair, the monad and the dyad; and the theological pair, Zeus the monad and the world soul, the dyad. Also, there is the connection between these two pairs: while the monad is the same in both pairs, the ontological dyad is different from the theological dyad.[18] This difference is also seen in the fact that the primal indefinite dyad is the unlimitedness, the infinity. As a material principle it can be considered the material for creating ideas and ideal numbers – which are infinite in number. It is also the origin of the material from which the world was molded. The world, in Plato's spirit, can be considered to be of finite dimension;[19] therefore, the world soul that is part of this created world cannot be infinite.

---

[12] And also to the origin of the soul as a mixture of the same, the different, and being (37a). Cf. also 50cd, 52c.

[13] A correspondence between tripartite ontological and epistemological division can also be found in Plato, *Tim.* 27d-28a, Detlef Thiel, *Die Philosophie des Xenokrates im Kontext der Alten Akademie*, München: Saur 2006, 251.

[14] Or the universe: in Plato, particularly in his late dialogs, the cosmos was synonymous with the universe, Kerschensteiner, *Kosmos*, München 1962, 226; Thiel, *op. cit.*, 248.

[15] Hans J. Krämer, *Der Ursprung der Geistmetaphysik*, Amsterdam: Grüner 1967, 34; Thiel, *op. cit.*, 246-247, 260, 262.

[16] 1028b24-27 = F29a = fr. 34; 1069a33, 1076a19 = fr. 34.

[17] This is a departure from Speusippus' view for whom *nous* should be distinguished from the one, i.e., the monad (F58).

With his views, Xenocrates tried to merge Plato's orthodox doctrine with the unwritten doctrines by merging the one and the Demiurge into one highest principle. The separatedness of the ideas and prime matter of the *Timaeus* is avoided by making the indefinite dyad their single source on which the Demiurge operated. The fact of continuation of Plato's theory was recognized by Aetius when he commented on the views he described that "the first part [i.e., the part about the highest God and the world soul, Xenocrates] considered after Plato" (fr. 15).

A consequence of Xenocrates' ontology can be found in his treatment of geometry, in particular, in addressing the problem of commensurability of geometricals. Plato in his last work condemned in strong terms as ridiculous and shocking ignorance the view that one length is always expressible in terms of another length, and similarly with surfaces and volumes (*Laws* 819de); the relationship of line and surface to volume are widely believed to be commensurable, whereas they are not, which, in Plato's opinion, is simply disgraceful not to understand (820ab). The denial of incommensurability would just be a step backward, and yet it is apparently a step taken by Xenocrates. It is stated that he believed in the theory of indivisible lines (fr. 43), and pseudo-Aristotle's treatise *De lineis insecabilibus* argues with an unnamed defender of just such a view. This defender was an academician since the idea of the line is used in one argument (968a9-14 = fr. 42), so it could not be Speusippus. It is usually believed that Xenocrates was meant here. One argument apparently tries to assure the commensurability of all lines. It states that all commensurate lines must be measured by some indivisible unit (968b5-15). If this unit is not applicable to incommensurable lines, then the entire argument is futile or at best proves that there are indivisible lines (that make up commensurable lines) and that there are also infinitely divisible lines (which are incommensurable lines). This would not quite work since line AB can be commensurable with some line CD and incommensurable with some line EF. Also, another argument states that a line that cuts a square unit would be neither rational nor irrational, and lines that produce rational squares would lack a determinate nature and, relatively to one another, they would be rational and irrational at the same time (968b17-22). The assumption of the existence of indivisible lines apparently solved the problem. Therefore, the likelihood that Xenocrates authored

---

[18] As stated confidently by Heinze, it requires no proof to see that the dyad as the highest principle and the dyad as the world soul are not the same, Richard Heinze, *Xenokrates*, Leipzig: Teubner 1892 [reprint, Hildesheim: Olms 1965], 35 note 1.

[19] There are eight gods, according to Xenocrates: five planets, the sphere of fixed stars, the sun, and the moon (fr. 17).

these arguments is great since it could not be Plato considering his rather disdainful position toward the deniers of the existence of incommensurability.[20]

Xenocrates' reason for admitting the existence of indivisible lines may be to bring geometry in line with his ontology. Everything that exists is the result of the one putting its stamp on the indefinite dyad. The indefinite dyad is indefinite since it is infinite and the action of the one is to, as it were, tame this infinity, streamline it by imposing confines on it, making it determinate and circumscribed. As to physical nature, the problem was solved by Plato when requiring elementary solids to compose the four elements, the solids themselves being constructed of elementary triangles (*Tim.* 53c-55c). Plato's physics is thus a form of atomism.[21] There remain geometricals which are infinitely divisible, and Zeno has shown to what problems this can lead (one of them is explicitly addressed in *De lineis ins.* 968a18-b5). Therefore, the introduction of indivisible lines apparently solved the problem, and geometricals, even if only noetic entities, truly become the product of the one and the indefinite dyad. Therefore, it can be stated that "there is something partless not only in the intelligibles but also in the sensibles" (968a17-18) and, presumably, the existence of indivisible units in the realm of intelligibles is ontologically prior to the existence of such units in the physical world. Xenocrates thus closed an apparent hole in Plato's system by requiring a clear impression of the one in the extraperceptible sphere at the cost of making his own version of Platonism to be out of step with contemporary mathematics.

The monad (the one) and the dyad of the unwritten doctrines, possibly espoused by Plato himself, indicate the growing importance of the problem of infinity for Plato and for the academicians. That importance is reflected in making the monad and the dyad the two highest principles at least by Speusippus and Xenocrates. Speusippus introduces five levels of being, and Xenocrates tries to make his system much closer to Plato's original doctrine, but for both scholarchs, the dyad, i.e., the infinity has a prominent ontological position as an ontological principle of all that exists. Truly, for the academicians, at the beginning was the *apeiron*.

---

[20] Therefore, Aristotle's ascription to Plato of the belief in the existence of indivisible lines should not be considered decisive, since "it is impossible to assume as Platonic the theory which is in flagrant contradiction with Plato's own words" expressed in the *Laws*, Zbigniew Jordan, *O matematycznych podstawach systemu Platona*, Poznań: Poznańskie Towarzystwo Przyjaciół Nauk 1937, 66.

[21] See a useful discussion of Plato's atomism in Eva Sachs, *Die fünf platonischen Körper*, Berlin: Weidmannsche Buchhandlung 1917, ch. 3.

13 THE EARLY LYCEUM

In the first book of *De caelo*, Aristotle stated that all bodies in the world move to their natural places in a straight line, if unhindered, upward or downward. There is also circular motion, natural and more perfect than rectilinear motion, and thus the body for which circular motion is natural must be more perfect than bodies subjected to linear motion. Such a body is neither heavy nor light, it is ungenerated and indestructible, i.e., eternal, not subject to increase or diminution, unaging and unalterable. As Aristotle observed, the existence of an element with such attributes was recognized by his predecessors as αἰθήρ (270b22), the fifth element, called by Aristotle the first body (284a30, 287a3). Aether is the material from which celestial bodies are built and also the cause of their circular motion. It also fills the entire celestial, supralunary region. However, in later books of *De caelo*, aether plays little role, and in *Physics* it plays no role at all; thus Aristotle himself pointed to the dispensable status of aether in the peripatetic universe.[1] A bit more decisive step was made by Theophrastus.

13.1 THEOPHRASTUS

Theophrastus, the successor of Aristotle at the helm of the Lyceum, was a prolific author; however, only a few of his writings are preserved. The extent of his interest in the problem of infinity is not exactly known. He wrote two books on Anaxagoras (DL 5.42), for whom infinity was of primary importance, and certainly in his discussion of Anaxagoras' views Theophrastus must have expressed his opinions on the subject, but the book did not survive. It is possible that in his *Physics* (DL 5.46) Theophrastus addressed the problem of infinity, continuum, and Zeno's arguments, but nothing is known today about it. Following in Aristotle's footsteps, Theophrastus addressed the problem of eternity: eternity of the world and eternity of the human mind, and his views on the subject have been preserved by ancient authors.

Theophrastus rejected Aristotelian dualistic cosmology, abandoned the idea of aether, and made his explanations in terms of two kinds of fire – which draws on Aristotle (*De gen. an.* 736b35) – stating that "the first sphere" is a pure form of fire and thus a pure form of heat which does not require any fuel to burn (*De igne* 4) and due to this heat plants can grow and animal life can flourish (*De igne* 5, 44). Pure warmth does not warm anything. The terrestrial fire, is mixed with fuel; the pure heavenly fire, including the fire of the sun, is unburning fire that requires no fuel (*De igne* 4-5).

---

[1] Friedrich Solmsen, *Aristotle's system of physical world*, Ithaca: Cornell University Press 1960, 299-303.

The expression "the first sphere" in *De igne* 4 seems to refer not just to the external limits of the cosmos, the sphere of fixed stars (cf. Aristotle, *Met.* 1073b18), but also to the entire supralunary region composed of aether because the problem of the nature of fire constituting the first sphere can hardly be limited to the external limits alone. The terrestrial sphere inside the first sphere is the entire sublunary region composed of the four elements.[2] The fiery character of celestial bodies was indicated by earlier philosophers, by Plato in particular.[3] Plato seems to have had only one kind of fire; Theophrastus stressed the existence of its two kinds. Aristotle criticized those for whom the heaven was pure fire (*Meteor.* 339b30) and those who saw stars as moving in fire (*De caelo* 289a16-19, 291a19-20). He also expressly stated that aether was not fire (270b20-25). For Aristotle, warmth and light of stars, of the sun in particular, was the result of friction of celestial elements moved in a circle against air (*De caelo* 2.7, *Meteor*. 1.3).

The sun's fine matter can reach the earth directly, contributing to the emergence of life (*De igne* 38, 44, 73) and to the incitement of winds (*De ventis* 15-16). This matter is specified as unmixed but also as mixed with air (*De igne* 4). Aether, on the other hand, was limited only to the supralunary sphere. However, the unburning character of pure fire makes it precariously close in nature to Aristotle's first body, differentiated from it merely by name. The reason for this similarity would be in the underlying idea of the eternity of the world. What guarantees this eternity? For Aristotle, the world is eternal due to the divine character of aethereal bodies that can move eternally without a need of influx of energy. The celestial bodies are eternal because of the nature of their material makeup, which is aether. The bodies need an incentive for their eternal motion and for the first heaven, this incentive stems from its desire for the unmoved mover, but Theophrastus simplified Aristotle's ontology and physics by making the first heaven self-sufficient in respect to its motion.[4] In this way, the role of the unmoved mover has little physical significance in the Theophrastean universe; however, the distinguished character of material constituting celestial bodies is still important. For Aristotle, aether somehow guaranteed that celestial bodies were able to rotate, and do it eternally. For Theophrastus, the purity of fire constituting these bodies apparently guaranteed that they could eternally desire rotational motion and be able to perform it, also eternally. This would solve the problem of eternity of the supralunary sphere for both Aristotle and Theophrasus, regardless of whether their argument can be found satisfactory: the heaven is eternal because it is made out of aether/pure fire, and the latter are eternal because they are, i.e., because this is their nature.[5] How is it with eternity of the sublunary world? The problem was apparently very important for Theophrastus since he addressed it at length as reported by Philo.

---

[2] Kaiser Gaiser, *Theophrast in Assos. Zur Entwicklung der Naturwissenschaft zwischen Akademie und Peripatos*, Heidelberg: Carl Winter Universitätsverlag 1985, 77.

[3] E.g., Anaxagoras, A71-84; Plato, *Tim.* 40a; Philip of Opus, *Epinomis* 981e; Xenocrates, fr. 56, Heinze.

[4] *Met.* 6a5-10, 10a9-21; cf. James Longrigg, Elementary physics in the Lyceum and Stoa, *Isis* 66 (1975), 218.

[5] The heaven is "in rotation according to its essence" (*Met.* 10a14).

Theophrastus addressed four arguments against the eternity of the world and rebutted them. The first argument from unevenness of the land states that the erosion caused by rains falling from eternity would eventually destroy even the highest mountains. Mountains obviously exist, *ergo*, the world is not eternal (Philo, *De aeternitate mundi* 23.117-119). Theophrastus responded with his theory of orogenic process: like trees, mountains also grow, although much more slowly; the fiery element from underground strives to its natural place and pushes upward some earthly substance in the process (25.132-137). The second argument, from the diminution of the sea, states that the sea is receding since there are islands where before there were none and the mainland increases in size by drying up bays (23.120-123). However, responded Theophrastus, many parts of the mainland were swallowed up by the sea and thus it is clear that the sea makes up for its diminution in one place by invading the land elsewhere.[6] The third argument is more general: the whole of nature is perishable if any of its parts is perishable, and any one of them eventually is, even the hardest of stones (24.124-129). Not so, said Theophrastus. The whole is perishable only when each of its parts perishes at the same time, but "if each [part] individually changes to the nature of its neighbor, it is made immortal rather than perishing," or rather, if some parts perish and other are generated to replace them, the whole remains intact.[7] The final argument claims that humankind should be eternal, but it is not as testified by the relatively recent emergence of human crafts (24.130-131). However, answered Theophrastus, cycles of floods and conflagrations destroy most of the human race, after which the remnant slowly rebuilds civilization.

The third proof is a generalization of the remaining three proofs,[8] and it seems clear that the eternity of the sublunary region is explained by the perpetual exchange of its elements when "each [part] individually changes to the nature of its neighbor." This is a perennial give-and-take process which, when done in proper proportions, guarantees that the whole exists eternally in the presence of incessant changes of its parts. It may be tempting to extend this to the supralunary region as well, which at least would give a measure of consistency in treating these two regions. A claim can be made that, for Theophrastus, the eternity of the world does not rely on the immutability of the supralunary domain due to the immutability of an eternal substance, but on exchange of forces and matter.[9] He did mention the fact that the sun exhausts the moisture in the exhalation.[10] It is true that sublunary fire feeds on moisture,[11] but this is hardly true about the sun, and

---

[6] 26.138-142, cf. Aristotle, *Meteor.* 351a19-25, 352a17-26, 352b16-19, 353a18-28.

[7] 27.143-144, cf. Aristotle, *Meteor.* 351a26-31, 357b26-358a2, *De caelo* 279b17-31.

[8] 27.145-149, cf. Aristotle, *Meteor.* 351b8-13, 19-28; Plato, *Tim.* 22c-23a, *Laws* 677a-d. See also John B. McDiarmid, Theophrastus on the eternity of the world, *Transactions and Proceedings of the American Philological Association* 71 (1940), 243-244.

[9] The claim is defended by Peter Steinmetz, *Die Physik des Theophrastos von Eresos*, Bad Homburg: Gehlen 1964, 31, 163-167, 327.

[10] *De ventis* 15; Steinmetz, *op. cit.*, 37-38, 164.

[11] *De igne* 10; Steinmetz, *op. cit.*, 122.

the statement is rather a description of the indirect influence of the sun and the direct action of the heat generated by the sun.[12] However, Theophrastus may have encouraged attempts of extending the same mechanism from sublunary region to the whole of the universe. The fact that he spoke about fire existing in the sublunary and supralunary regions is significant since it expressed a desire to have more integrated physics than there was in the case of the Aristotelian universe. And, as mentioned, particles of the sun do reach the lower strata of the sublunary region which may make one wonder whether it may lead to exhaustion of the matter of the sun. However, Theophrastus may have defended the eternity of the sun by stating that fine particles coming from it do not mix with anything and eventually return to it, thereby guaranteeing the integrity of the sun from eternity to eternity. One thing is certain: the doctrine of temporal infinity of the world remains the cornerstone of and an axiom for the systems of Aristotle and of Theophrastus. Both of them defended this doctrine, pointing to the fact that the finite existence of elements of the world did not undermine the eternity of the whole, and both of them defended the view that what is eternal takes precedence over what is corruptible (*Met.* 4a16-17, *Met.* 1050b6-7). Also, a more prosaic reason for eternity of the world may have been, as illustrated by adherents of this view: it is impossible to decide whether eggs or birds came first since one cannot exist without the others (Censorinus, *De die natali* 4.2-4).

13.2 EUDEMUS

Eudemus, a contemporary of Theophrastus, was Aristotle's pupil and in the opinion of Simplicius, who frequently quoted him, Eudemus understood the thought of Aristotle better than all other exegetes (*In Phys.* 991.28-29 = fr. 105) and was a lover of truth (1024.7 = fr. 108). In his *Physics*, now lost, he discussed the problem of infinity. However, as can be determined from the preserved fragments, Eudemus did not go in his views beyond his master. He stated that time and change are continuous and have parts, i.e., they are divisible because distance is continuous and divisible (fr. 85), which has already been more succinctly said by Aristotle (*Phys.* 219a10-14).[13] He agreed with Aristotle that "to say that what is of the same form is infinite in quantity is not different from its being infinite in magnitude" (fr. 62), that is, an infinity of bodies similar in form would require infinite space to accommodate.

Aristotle listed five reasons for the existence of the infinite (*Phys.* 203b15-25); Eudemus claimed to have added one more reason. For both of them, one reason is infinite divisibility of the continuous, to which Eudemus added infinite counting as a counterpart of the division process (fr. 64). But, as Simplicius remarked, Aristotle did not omit this reason when listing infinite things through generation when he said "number also seems to be infinite" [203b24] (Simplicius, *In Phys.* 468.7-10). The two processes are identical in nature (Aristotle, *Phys.* 203b3-4); the additional argument is legitimately not listed by Aristotle.

---

[12] R.W. Sharples, Theophrastus on the heavens, in J. Wiesner (ed.), *Aristoteles: Werk und Wirkung*, v. 1, Berlin: de Gruyter 1985, 580. Although for Aristotle the sun as made out of aether could not warm anything (*Meteor.* 341a32) he spoke about it as though it did (341b7-8).

[13] Simplicius contrasted Eudemus' view with Strato's according to whom continuity of motion does not depend on continuity of distance (magnitude); motion is simply continuous in itself (Strato fr. 70).

One of the most interesting arguments in the discussion of the problem of infinity mentioned by Eudemus is not even his own, but it was repeated after Plato's friend, Archytas. The thought experiment proposed by Archytas was meant to show that the universe is infinite since, if it were finite, a person could stand at the boundary of the universe and stretch his hand outside – and to Archytas it would be absurd to assume that this would be impossible – and outside would be either body or space. Simplicius only reported slight elaboration of this argument by Eudemus but not his counterargument (467.26-35 = fr. 65 = 47A24). Incidentally, the argument was very appealing and many of its versions were later used by various philosophers.[14]

13.3 STRATO

Strato, the third head of the Lyceum, is said to have "abandoned the most essential aspects of philosophy, values, and ethics, and devoted himself entirely to the investigation of nature. In this respect he differed most of all from his own descendants" (fr. 13). And, in fact, the preserved testimonies seem to confirm his interest primarily in natural philosophy to the exclusion of other fields.

His view that the heaven is fiery (fr. 84) amounts to discarding the fifth element, aether, and abolishing the division of the cosmos into the supralunary, aethereal realm, and the sublunary region. The world is made from the four elements – air, water, earth, and fire. This simplified Aristotle's physics, but did not have to necessarily endanger the eternity of the world: the four constitutive elements can just as easily be assumed to be eternal substrates.[15]

It seems that Strato assumed with Aristotle that the world was uncreated. He said in his discussion of time that everything that is extended, is in time (fr. 81); "if to be in time were to be encompassed by time, then it is obvious that nothing eternal could be in time" (fr. 80), which is in contrast to Aristotle's statement that "things which always exist are not in time, for they are not contained by time" (*Phys.* 221b3-5). That is, for Strato, the eternal things are not encompassed by time although they are in time,[16] which very likely means that there exist eternal things – the world as a whole or at least the qualities that are the substrate of the things that exist are eternal. Infinity of existence is not constrained by time, even an infinite time;[17] existential infinity is not restricted by temporal infinity, the latter is, as it were, more limited than the former, existential infinity is more pure, less constrained, more general than temporal infinity. Actually, there may be even doubtful whether eternal time in pure form exists since "something always existing in time [in reality] exists between that which is before time was and that which at some point [of time] exists" (fr. 127). Temporal *eternity*, to be eternal, must include pre-temporal existence. Temporal infinity reaches beyond time. Existential infinity is very close to infinity *tout court*, infinity in pure form, infinity as the principle of being that assures that the world exists, regardless of all its vicissitudes – eternally.

This may not be easy to reconcile with the nature of the eternity of the world. In what sense is the world eternal if eternity appears to be of an extra-temporal character? Could it be that there is an ingredient of Eleatic thinking in a peripatetic philosopher? Or maybe it is an element of Platonic thinking where time is merely a moving image of eternity? Strato would be hard pressed with his materialist thinking to defend extra-temporal eternity and there is no indication that he ever found it problematic.

With the eternity of the world, Strato saw no need for the first mover: the world has always been in motion, the causal chains are extended infinitely into the past, and because no final causality exists, the past eternity of the world solves the problem of the beginning of motion. And why would the world be in motion rather than at rest? Because this is the way the world is and the causes of motion are within it. They are blind, unreflective causes, lacking all sensation – unguided causes, not directed, not molded by any providential force, and as such they simply lack shape (fr. 33).

In respect to the spatial dimension of the world, it seems that according to Strato the world is very likely finite since he seems to have spoken about the center of the world.[18] However, there is a testimony suggesting that the world may be infinite (fr. 48),[19] and nothing in his physics would contradict such an attribution, but it is possible that Strato referred to infinite divisibility.[20] This means that Strato did not overcome Aristotle's reluctance to see the world as spatially infinite.

---

[14] Richard Sorabji, *Matter, space, and motion*, Ithaca: Cornell University Press 1988, ch 8.

[15] Matthias Gatzemeier, *Die Naturphilosophie des Straton von Lampsakos*, Meisenheim/G.: Hain 1970, 57.

[16] "Just as the trouble is in the city, not city in trouble, so time is in things, not things in time," as phrased by G. Rodier, *La physique de Straton de Lampsaque*, Paris: Alcan 1890, 77. Cf. Gatzemeier, *op. cit.*, 127-128

[17] If there were time that encompasses eternity, "there would exist time that transcends eternity," Gatzemeier, *op. cit.*, 128 note 12. When criticizing being in time, "his point may have been that the lifetime of anything is governed by its 'being' in the sense of its nature or intrinsic structure," Hans B. Gottschalk, Strato of Lampsacus: some texts, *Proceedings of the Leeds Philosophical and Literary Society, Literary and Historical Section* 11 (1965), 168.

[18] According to fr. 90, the Earth is immobile, which suggests geocentrism, although Diels advocated heliocentrism as Strato's view basing his contention primarily on the testimony that the stars are illuminated by the Sun (fr. 85), which presumably points to the Sun's central position in the universe, Gatzemeier, *op. cit.*, 56; Rodier, *op. cit.*, 87 note 4.

[19] Spatial infinity of the world was attributed to Strato by C[arl] Nauwerck, *De Stratone Lampsaceno philosopho disquisition*, Berlin: Reimer 1836, 29.

[20] Rodier, *op. cit.*, 79; Fritz Wehrli, *Die Schule des Aristoteles*, Basel: Schwabe 1969, v. 5, 56.

In sum, Aristotle's view of infinity of the world was in essence maintained by his successors in the Lyceum. The world was finite spatially but infinite temporally or rather – considering Strato's view – of infinite duration. Also, the view of infinite divisibility of matter and space also remained uncontested (Strato, fr. 82), but, at least for Strato, the view was not extended to time which, in his opinion, consisted of the extended nows/instants (fr. 82) so that time was continuous (fr. 75), but not infinitely divisible.[21]

---

[21] Cf. Ernst G. Schmidt, Straton-Zitate bei Damaskios, *Museum Helveticum* 19 (1962), 220-221.

14 EPICURUS

Epicurus concluded his letter to Pythocles with the urging that Pythocles should above all things give himself up "to the study of the principles and of infinity and of the things akin to them" if he wanted to reach the primary goal (Diogenes Laertius 10.116) which is freedom from disturbance and certainty of conviction (10.85), i.e., happiness (10.128, 131). That is, the study of philosophy would be redundant if fear of death and fear of natural phenomena would not threaten us (*Basic doctrines* 11, 12, 10.142-143). Study, in particular, the study of infinity as the way to happiness, certainly commands attention to the problem of infinity.

14.1 ATOMS AND THE VOID

Epicurus' ontology is based on an oft-repeated principle that nothing can come from nothing (10.38; cf. early atomists 68A1.44); therefore, the universe is eternal, i.e., of infinite duration, and, as a whole, immutable – it was, is, and will always remain the same (10.39). That is, no new entity comes into being which is not derivable from already existing entities, which are eternally existing and unchangeable atoms, and the void.

There are only atoms and the void; everything else must be an accident or a temporary property, or a characteristic feature, permanent property of bodies or void (10.40); i.e., primary properties – size, shape, weight, color, taste, sound, smell (Lucretius 2.381-477) – are not just conventions, as for Democritus. They are real.

Epicurus called the void space and intangible nature (DL 10.40), place and intangible substance (SE 10.2, Aetius 1.20.2).[1] The void, or, more generally, the universe (τὸ πᾶν), is infinite since if it had been finite it would have had a limit and there would be something else than the void and atoms (10.41).[2] Therefore, whatever exists, must exist inside the void; whatever exists, must be extended in space (10.67). A javelin thrown by someone standing at the edge of the universe would be stopped by something else or it would go on; whichever case it is, there is no edge (Lucretius 1.968-983). Simplicius stated that according to some atomists, the void exceeds bodies in infinity (*In Phys.* 618.16-18). Since there are infinitely many atoms in infinite space, this remark may mean that the void is not tightly packed with atoms and there is enough room for infinitely many atoms.

---

[1] Identification of the void with place was made under the influence of Aristotle: Friedrich Solmsen, Epicurus on void, matter and genesis, *Phronesis* 22 (1977), 264-265; Brad Inwood, The origin of Epicurus' concept of void, *Classical Philology* 76 (1981), 273-274. See also David Sedley, Two conceptions of vacuum, *Phronesis* 27 (1982), 183-191.

[2] For Aristotle, finite universe was not limited by anything. It seems that he meant Epicureans in his mention of this proof in *Phys.* 206b33-207a2. Cf. Cicero, *De div.* 2.103, Lucretius 1.958-964. A similar proof was used by the Eleatics (Aristotle, *De gen. et corr.* 325a14-16 = 28A25); Ivars Avotins, On some Epicurean and Lucretian arguments for the infinity of the universe, *Classical Quarterly* 33 (1983), 424.

Because the void is infinite, there must exist an infinity of atoms since, otherwise, they could not stop in their motion through space (DL 10.42). By itself, the argument is not convincing: an infinity of atoms could go eternally through infinite space without being stopped by anything. An infinity of atoms by itself does not explain the existence of conglomerates of atoms and is not implied by the infinity of the void. This is an independent assumption which can very naturally be made about the infinite space, although if no lower limit had been set on the sizes of atoms, an infinity of atoms could also be accommodated in a finite universe. However, Epicurus very likely would simply find such a possibility to be incomprehensible just as he found incomprehensible an infinity of atoms forming a finite body (10.57). Therefore, Epicurus imposed a lower limit on the size of atoms (10.59; Lucretius 1.748-752). For him, infinity of atoms cannot form a finite body, which, he argued, would be impossible even if atoms could have any size (10.56), which, of course, does not follow since, by Zeno's argument, an infinity of atoms can form a finite body if no limit on their sizes is imposed. Maybe Epicurus sensed this after all, and that is why he imposed such a limit. Moreover, the infinite divisibility of atoms should be rejected to avoid the weakening and even the destruction of bodies (10.56). However, the indivisibility of atoms does not have to contradict the assumption of the existence of atoms of any small size below a certain upper bound. The argument must be that somehow small atoms are not as good to generate durable conglomerates as larger atoms. Actually, the opposite argument could be made: small atoms could generate more durable bodies because their motion would be weaker than the motion of larger atoms and thus the integrity of the entire body would be stronger since the motions of individual atoms would not attribute to the disintegration of the body as strongly as the motions of larger atoms. Also, because atoms would be more numerous, it would be more likely that opposite motions of individual atoms would cancel each other, thereby enhancing the integrity of the body they compose.

Not only a lower limit, but also an upper limit on the size of atoms is imposed: an absolute perceptibility limit (DL 10.55-56; Lucretius 1.265-270, 2.479-480). Therefore, there cannot be atoms of the size of a world perceptible in one world and imperceptible in another, as permitted by Democritus.

The sizes of atoms vary between an upper and a lower limit, whereby Epicurus avoided first, a counterintuitive possibility of atoms being not only perceptible but also as large as a world; and second, a possibility of matter to be dissipated into nothingness – or rather into points of no size, which, in his mind, would be tantamount to annihilating matter into nothingness. But still the problem raised by Zeno – the problem of passing an infinity of points by a moving atom – can reappear: if atoms between the imposed limits can have any size, it would mean that space, at least between these two bounds, could be infinitely divisible because each length between the two bounds would have a physical representation in an atom. Only one step remains to reproduce Zeno's paradoxes in the atomist framework. To avoid it, Epicurus imposed the limit on the variation of sizes in general

and of sizes of atoms in particular by introducing minimal parts out of which atoms can be composed. Minimal parts are solidly blended together so that breaking down an atom into these subatomic particles is physically impossible and can be done only theoretically (DL 10.56-59; Lucretius 1.599-634, 746-752, 2.478-499). The solidity of blending of minimal parts in fact guarantees that atoms are eternal (Lucretius 1.503-539, cf. 67A13, 67A14). But, more importantly, the minimal parts seem to be all of the same size and thus their size becomes the yardstick of measurements, minimal units of magnitude – of bodies and of space (DL 10.59). After imposing granularity of material composition of atoms, granularity of space is established, or the former is introduced so that the latter becomes possible. When space becomes granular, Zeno's quandary is no more.

We should not think, argued Epicurus, that atoms can be of any size, but they do have different sizes (DL 10.55). There is an ungraspably (ἀπερίληπτα) large, but not infinite, number of shapes since all the phenomena cannot be explained by a limited number that presumably can be grasped by the mind (10.42).[3] But the number is limited (Lucretius 2.478-499) and some shapes are excluded (DL 10.74) since atoms of these shapes would be breakable (Aetius 1.3.18). There are, however, infinitely many atoms of each shape (10.42), otherwise, he argued, the number of atoms would be limited (Lucretius 2.522-531, 1.1008-1013). Of course, for the latter to be true, an infinity of atoms of just one shape would be sufficient, but then the problem would be, which shape it should be. Therefore, in order not to favor any permitted atom shape, all of them are declared to be represented in an infinity of atomic embodiments. Also, the principle of *isonomia* requires an equal balance between atoms of different shapes, and thus quantities of atoms of different shapes have to be comparable.

Epicurus insisted that atoms move downward in the void, and yet the void is infinitely extended in all directions. The natural question is, what is down and up in the void? Epicurus was aware of the problem (DL 10.60), but did not provide a satisfactory answer. If space is assumed to be isotropic then no satisfactory answer can be really provided. In fact, in the isotropic space, there is no real reason for any motion at all. The fact that Epicurus recognized weight as a primary property of atoms does not help much either since weight could be used to explain a motion in a direction, any direction, not down where there is no down. In that case, when each direction would be equally likely for the motion of an atom, collisions would easily occur and there would be no rationale to introduce the swerve as a means to break from the absolute downward movement.

Downward movement could be explained by its eternity: atoms are in eternal motion (10.43) and there is no point to ask how downward movement started; it can simply be accepted as an eternal fact. Atoms move in one direction because they have always moved there and this direction is therefore considered downward. The fact of motion would not be explained by a theory of natural motion, but the latter would be enforced by the former. This explanation does not hold, however, after atoms collide with other atoms – atoms are thrown away from their downward motion, but later, after the impact of collision subsides, they resume their downward motion. Therefore, only one possibility seems to remain: Epicurus did not consider space to be isotropic. For him, there was an absolute space with absolute directions because of the anisotropic nature of space.[4] In this way, Epicurus would extend Aristotle's theory of natural movement to an infinite universe. And it just so happens that the up and down directions are distinguished with respect to the earth (10.60), although the earth is not the center of the universe (Lucretius 1.1052-1082).

---

[3] Which was the restriction made very likely as the result of Aristotle's criticism, David J. Furley, Aristotle and the atomists on infinity, in I. Düring (ed.), *Naturphilosophie bei Aristoteles und Theophrast*, Heidelberg: Stiehm 1969, 92.

All atoms fall downwards at the same speed. Bodies can be formed when atoms spontaneously swerve from their courses, thereby colliding with other atoms.[5] Generation and destruction existed from infinite time (Lucretius 5.187-191), therefore, there does not appear to have been time when atoms were not falling in the void and when they started swerving giving rise to single objects and entire worlds.[6]

Since the void poses no obstruction for moving atoms, the atoms should move with infinite speed and thus imposing finite speed through the void is arbitrary.[7] With such speeds, atoms would move from one point to another in no time, or rather at time points; however, motion is observed in the world, so that Epicurus extended atomistic approach from matter also to time and space: there are minimal distances and minimal time periods and minimal motions (Simplicius, *In Phys.* 934.23-30; Themistius, *In Phys.* 184.9-28; spatial atoms are suggested in DL 10.59, temporal atoms in 10.62; Lucretius 4.794-796). This solves the problem of infinite speed if an additional restriction is imposed: atoms can cover any number of spatial atomic intervals in one temporary atomic interval (cf. DL 10.46), which probably resulted for Epicurus in assuming the existence of atomic motions.[8] Motion would be performed by quantum leaps: an atom jumps from one end of a spatial atom to another without actually passing its intermediate parts. This could happen if the moving atom disappears at one end of the spatial atom and reappears at its other end, which would turn motion into incessant acts of creation and recreation of moving material atoms in the spirit of continuous creation proposed later by Descartes.[9] However, there is no creator in the Epicurean universe; therefore, to avoid paradoxes of motion in atomic time through atomic space, the requirement can be made that after motion begins at one end of the spatial atom, this motion can end only when the moving atom reaches the other end of the spatial atom. The whole of space would be like a grid and an atom can only move by being unsnapped from the grid and being snapped back to it at another point of the grid, i.e., the moving atom cannot stop between grid planes since if it tried, it would slide into the nearest grid-plane.[10] Therefore, intermediate parts of spatial atoms would exist, i.e., spatial points could be assumed to exist,[11] but they would be out of reach as stopping points for moving atoms.

The introduction of spatial atoms, however, poses more problems than it solves. If space is a grid of spatial atoms, the only motions that are possible seem to be vertical and horizontal motions. Can an atom move at an angle? One solution would be that the spatial grid is in relation to the moving atom; therefore, there would be infinitely many grids in space with different angles in respect to one another. But if an atom moves by one spatial atom to the left and another atom moves one spatial atom 45° in respect to the trajectory of the first atom, the two atoms will be at a distance less than one spatial atom from one another after their respective moves. Another solution can be to require one spatial grid only, but if an atom moves at an angle, it can be unsnapped and snapped back only in the grid. In this way, when an atom moves from one corner of the spatial atom to the opposite corner, it covers the distance $s\sqrt{2}$ that is longer than the length s of one side of the spatial atom and yet it would be considered an atomic movement. In this way, a quantum leap would be done for distances between s and $s\sqrt{2}$, which would introduce an unwelcome fuzziness into the concept of minimum distance. Another problem is the motion by an angle other than 45°. If other angles are permitted, a cubic atom of size s^3, after passing across a spatial atom, would stop between planes of the spatial grid, and it should fit exactly into a spatial cube. Therefore, all motions would be jerky movements, say, one spatial atom up, two spatial atoms to the left, one spatial atom up, two spatial atoms to the left, etc., thereby creating an uneven line like jagged lines on the computer screen that can never be perfectly smooth. The same would be for trajectories other than straight lines. And how about rotation? Could an atom rotate by any angle that is not a multiple of the right angle? If so, the atom would be misaligned with the grid. But obviously we observe rotations of bodies by any angle. How do we reconcile that with the existence of minimal spatial atoms?

Finally, the problem of the shapes of atoms. Epicurus permitted only a finite number of shapes, but did not address the problem of shapes themselves. With the presence of spatial atoms it seems that the minimum component that could be used to form an atom should be a cube. If this component were a cylinder inscribed into a spatial atom, there would be a question of the circumference of the circular side of the cylinder, which, being $2\pi s$, would not be a multiple of an atomic spatial interval. However, the same problem can be raised in respect to the minimal cube. Its side would be of length s, but measurement between any two points on the edges of one side which are connected by a line not parallel to two edges of the side would also be slightly larger than s. The problem can simply be solved by saying that surface would be considered part of what is already considered the minimal part, thus surface or the distance between any of its two points cannot be measured because no points exist, either. But the problem may reappear on the macro level when taking a measurement of, say, a diagonal of one side of a macrocube. The measurement could not be expressed perfectly in terms of minimal intervals, and the problem can be dismissed by saying that, at the macro level, perfect precision is unattainable and also unnecessary, so rounding to the nearest multiple of atomic interval would suffice. After all, even today, floating-point numbers used by computers only approximate real numbers. However, how small could measured objects be to accept such rounding? Although nanotechnology was not Epicurus' problem, the sizes of objects that can be handled could be of atomic size and then the rounding error could near 50%. Maybe precision should be left to geometry and imprecision could be endured in the physical world because of the way the world is. But Epicurus had little patience for mathematics. His education in the field, if any, was not impressive (Cicero, *De fin.* 1.71-72; SE 7.1; Plutarch, *Non posse* 1094d). He actually rejected geometry as false (Cicero, *Acad.* 2.106), and Epicureans in general proposed to "overthrow the principles of geometry" (Proclus, *In Eucl.* 199.9-10), which would mean that there is no room for perfect precision in the Epicurean outlook.

---

[4] "Epicurus believed intuitively in an absolute frame of reference, a fixed spatial grid, in terms of which falling atoms had real motion and no atoms could be said to be at rest," David Konstan, Problems in Epicurean physics, *Isis* 70 (1979), 416; David Konstan, Epicurus on 'up' and 'down' (Letter to Herodotus § 60), *Phronesis* 17 (1972), 277.

[5] Plutarch, *De sollertia animalium* 964c; Lucretius 2.216-293, Aetius 1.12.5, 1.23.4, Cicero, *ND* 1.70.

[6] "The priority of downward fall of atoms to the swerve is a logical priority only," J.M. Rist, *Epicurus: an introduction*, Cambridge: Cambridge University Press 1972, 51.

[7] Elizabeth Asmis, *Epicurus' scientific method*, Ithaca: Cornell University Press 1984, 282.

[8] Ontologically, however, motion precedes time; it is a property of time and time does not exist *per se* (Lucretius 1.459, SE 10.219). Cf. Aristotle, *Phys.* 231b21-25.

[9] Cf. DL 10.62. Themistius ascribed to Epicurus what is, in effect, only a verbal solution: Epicurus did not say that an atom moves (κινεῖται) over an atomic interval, but that it has been moved (κεκίνηται) (*In Phys.* 184.13); cf. Aristotle, *Phys.* 231b20-30, 232a7-8, 241a1. Merlan defended this view only by quoting Poincaré, Whitrow, and Grünbaum, Philip Merlan, L'univers discontinu d'Épicure [1968], in his *Kleine philosophische Schriften*, Hildesheim: Olms 1976, 327-332.

[10] "A fixed spatial grid" is mentioned by Konstan, *op. cit.*, 414, 416 (he also claims such a grid for Democritus, p. 402 note 25); cf. David J. Furley, *Two studies in the Greek atomists*, Princeton: Princeton University Press 1967, 116.

[11] Continuity of temporal and spatial atoms is considered to be a reasonable assumption by Asmis, 283.

If we assume that minimal parts are cubes, all atoms can only be made out of perfectly aligned cubes. If one atom is of the size s^3, another atom cannot be a half of it since it would be smaller than the spatial minimum. If two cubes are put together to form an atom, one cube cannot protrude beyond another cube since this would create a step between these two cubes with the length smaller than the permitted minimal distance. Therefore, atoms are made out of cubic Lego blocks which can be connected only when a side of one block perfectly aligns with a side of another block. What about spherical atoms which were so important to Epicurus at least in the context of discussion of the soul? The spherical shape of a spherical atom could only be approximated by a configuration of minimal cubes, which hardly would give them the smoothness required by Epicurus. Can an atom be a perfect sphere? If it were, it would have to be a configuration of minimal parts. If we envision such an atom in the spatial grid, a problem with outermost parts of the sphere marked by the grid immediately appears since many – if not most – of these parts would be noncubes with one rounded surface and, most importantly, all three dimensions would be smaller than the minimum distance. To save Epicurus' conclusions concerning the activity of the soul, the only spherical atoms that could be permitted should be of the minimal size.

14.2 INFINITE WORLDS

Following the early atomists, Epicurus also allowed for an infinity of worlds in infinite space because there is an infinity of atoms and it is impossible for all of them to be used up in creating just one world or a finite number of worlds whether they resemble our world or not (DL 10.45, 89). The reason probably was that this infinity of atoms would create an infinitely large world which somehow was inconceivable to Epicurus. Also, these atoms would congregate together, leaving the remaining parts of space truly void, which also was inadmissible to Epicurus.

The worlds are solely the result of the motion of atoms and there is no design behind their emergence and dissolution (Lucretius 1.1021-1028). Each world eventually perishes, no single world is eternal (DL 10.73) and atoms freed by the perished world will be used in creating another world. Epicurus derided the vortex mechanism used by his atomistic predecessors, but proposed in its stead a mechanism which hardly can be reduced to mere randomness of atomic motions. He stated that proper atoms – properly shaped and sized – have to come together and become properly positioned, properly connected together, and properly reconfigured when new atoms arrive from space (10.89). Epicurus' trust that massively large amounts of randomly moving atoms can give rise to such favorable conditions – and infinitely many times in infinitely many parts of the void – only stretches the imagination and exposes Epicurus to the same criticism to which he subjected the early atomists. But it certainly indicates Epicurus' unwavering faith in the power of the infinite: an infinity of atoms in infinite space and infinite time just has to produce – infinite times – highly organized results, the entire *kosmoi*.

Due to such random origin, worlds are never the same; however, the differences are rather small since more features are the same in different worlds than they are different: all worlds are portions of the sky that include the stars, the earth, and all phenomena that we know in our world. Each world is surrounded by a membrane of various consistency and thickness. Basically, the worlds can be of different sizes and shapes (DL 10.88), although not all shapes are possible (10.74). Presumably, like in the case of atoms, some shapes could render worlds easily breakable, and thus unstable. There is no limit to the kind of life that can emerge (10.74). Presumably, being alive, i.e., possessing a soul guarantees that organisms can retain their integrity for a nonephemeral amount of time even when they are monstrous combinations of body parts (Lucretius 5.836-854).[12] In sum, Epicurus allowed for infinite variations of infinite worlds; however, the variations are severely circumscribed in comparison to the variability of worlds in the early atomism. Each Epicurean world consists of basically the same elements (stars, planets, animals, etc.); Democritean worlds, on the other hand, could be widely different: with animals or without, with the sun or without, etc.; that is, the Democritean view is more true to the spirit of randomness saturating atomism. Creating basically the same world structure infinite times by the random movement of atoms as proposed by Epicurus makes his conviction well nigh impossible to accept. Cosmological mechanism is different in Epicurus and his predecessors; however, the mechanism for all the atomists presumably relies on the random motion of atoms and their properties, which appears to be hardly possibly when more closely probed. This faith in the self-organization of atomic matter rests for all atomists on the infinite: sheer amount of atoms, sheer expanse of space and time should by itself lead to generation of worlds.

---

[12] Incongruously, mythical creatures could never become real (Lucretius 5.878-924). Also, this process of relative stability may exclude plants since they were not considered living beings; their motion was considered automatic or mechanical, not under the control of the soul (Aetius 5.26.3).

14.3 KNOWING THE INFINITE

Epicurus wrote that the knowledge of natural phenomena contributes to happiness, knowledge that goes to basic principles; otherwise, partial knowledge can be more detrimental to happiness than blissful ignorance (DL 10.79). Is such knowledge and thus happiness attainable? He stated that the number of atom shapes is finite and yet ungraspable (10.42), i.e., even the finite, if sufficiently large, cannot be grasped by reason. On the other hand, knowledge of infinity is necessary for the attainment of happiness. How can such knowledge be attained if reason falters already in the domain of the finite? The situation is not hopeless since the inability to grasp does not necessarily mean that we are altogether out of our depth.

According to Epicurus, "beyond these two [bodies and the void] nothing can be thought either by grasping (περιληπτικῶς) or by analogy with graspable things (ἀναλόγως τοῖς περιληπτοῖς) such as they could be grasped (λαμβανόμενα) as whole existences," i.e., as independent entities (DL 10.40). Also, the movement of atoms through the void "accomplishes every graspable (περιληπτόν) length in an incomprehensible (ἀπερινόητος) time" (10.46). On the other hand, the time of atom motion in conceivable only by reason (λόγῳ θεωρητὸς χρόνος); this short time is beyond the range of perception and is conceivable only by reason (διὰ λόγου θεωρητὸς χρόνος) (10.62). There are apparently different levels of understanding. The shortness of time during which atoms move cannot be grasped and yet it is conceivable. It is possible that by grasping Epicurus understood a mental act which is directly connected with some visual component: with images arriving through sensory perception or images created by the mind itself.[13] This grasping would basically be visualization. Conceiving by reason, on the other hand, would be performed with a purely abstract concept which cannot be grasped or visualized. For this reason, Epicurus could say that because observation is impossible, by reason's contemplation (διὰ λόγου θεωρία) we also accept minimal length units for atoms (10.59). Atom sizes are ungraspably small which by itself does not devoid them of reality. They exist, but they can only be conceived by reason, not grasped by the mind. The speed of atom and the concept of infinity also belongs to this category. Infinity cannot be grasped; it cannot be pictured even by the mind's eye, but it is a rational concept which must be assumed to make sense of all perceptual data and make physics and ontology coherent. Infinity, atoms, and the void are, as it were, postulates of theoretical reason in Epicureanism. This does not mean that anything goes, since perceptual data and concepts all have to form a coherent whole. Therefore, Epicurus could state that the moving body does not arrive at different places at the same time in a minuscule amount of time conceivable only by reason (διὰ λόγου θεωρητὸς χρόνος) – for this is inconceivable (ἀδιανόητον) (10.47). Or stronger yet, this is rationally inadmissible since it is in contradiction with everything we know. Also, as Epicurus argued against the incorporeality of the soul, because we have to clearly comprehend (προσκατανοεῖν) that the incorporeal is what can be thought (ἐπὶ τοῦ … νοηθέντος) as such as an independent existence, it is impossible to think (οὐκ ἔστι νοῆσαι) about anything incorporeal outside of space, except the space itself, i.e., the void (10.67); because space encompasses everything, it would be contradictory with its nature to assume that there is something outside of space; it is even impossible to think about such an entity on the most abstract level. But perhaps Epicurus was not altogether consistent with himself. He stated that the reliance only on grasping, only on περίληψις, created a hurdle in envisioning the possibility of an infinity of atoms forming a finite body (10.56) and it is impossible to think (οὔτε … ἔστι νοῆσαι) how an infinity of atoms of any size could form a finite body (10.57). Maybe reliance only on *logos* would not face this difficulty if *logos*, as suggested above, would use Zeno's argument to deal with atoms of any size in a finite body.

The attainment of happiness requires a strenuous mental effort to contemplate most theoretical principle of the universe; we should commit ourselves to constant meditation of the all and of the highest principles (DL 10.82). Only a theoretician – a philosopher, a wise man – can be happy since he reaches the level of abstraction which reveals the mysteries of the universe; that is, the physical makeup of the universe on the most fundamental level, and Epicurus is convinced that happiness inevitably follows from this knowledge. The mind (διάνοια) should free us from the fear of eternity (*Basic doctrines* 20, 10.145), but this involves the contemplation of infinity – temporal infinity or, more generally, infinity in respect to duration. Knowing spatial infinity can presumably help to dispel the fear of natural phenomena and thereby make humans happy. Happiness through knowledge of infinity. Although it may be unclear how this knowledge can engender happiness, it is important to see that Epicurus endows this knowledge with personal dimension. It is not only interesting to know the foundation of the universe, but this knowledge brings fulfillment in the form of the personal happiness. If for no other reason, this is why, according to Epicurus, philosophy should be pursued.

---

[13] Thought is the grasping (περιλαμβάνειν) of a visual image (εἴδωλον). This can be πρόληψις created by the arriving εἴδωλα (περιληπτικῶς), sometimes it is an image created by the mind (ἀναλόγως τοῖς περιληπτοῖς), Cyril Bailey, *Epicurus: the extant remains*, Oxford: At the Clarendon Press 1926 [reprint, Hildesheim: Georg Olms 1970], 182; see also Asmis, *op. cit.*, 281.

15 THE EARLY STOA

There are two general categories in the Stoic ontology: something and non-something. For Plato, to be something is already to exist (*Parm.* 132bc). However, the Stoic category of somethings is divided into 1. bodies, 2. incorporeals (*lekta*, void, place, time), and 3. entities "falsely formed by thought" (Centaurs, Giants), and mathematical constructs (points, lines) which can be considered as neither corporeals nor incorporeals.[1] On the other hand, there are concepts that correspond to Platonic ideas; they are non-entities, non-somethings, at best quasi-somethings or "figments of the soul" (1.65; 2.329; DL 7.60-61)[2] since in the Stoic ontology there are only single, individual entities, whether corporeal or incorporeal (2.361).[3] The problem of infinity arises, therefore, in two areas, in the realm of bodies and in the realm of incorporeals.

Zeno of Citium says that substance is finite (*essentia finita*, 1.88), and the Stoics in general say that prime substance is limited or finite and God made the cosmos and every item in it from this limited prime matter and prime substance (2.323). This situation never changes. The volume of the world may change, but "the eternal substance becomes neither more nor less," thus it is always finite (1.87). There is only one cosmos "for all the bodies are included in it," and it is finite (2.528, 534; 1.97). The world is not only one and finite, but, as Posidonius later claimed, also spherical (DL 7.140). How can infinity be fitted into an ontological and epistemological image in which the world is inherently finite?

15.1 THE VOID

According to Zeno, the world is ultimately built out of two principles – passive matter and active God – and four elements – fire, air, water, and earth. Chrysippus retains only the four elements and transfers to fire the characteristics of the designing fire which is Zeno's active principle. However, both Zeno and Chrysippus agree on one point: at prescribed times, the world is destroyed and then recreated anew by God. During this cosmic conflagration, the whole of matter is turned into the passive principle, according to Zeno, or into fire, according to Chrysippus. From observation it was clear that fire occupies more space than fuel, so, by extension, the flame resulting from world conflagration must occupy more space than the four-element world before the conflagration. Therefore, there must be some room "more than ten thousand times greater" where the world can expand when vaporizing (2.537). This room the Stoics consider to be the void.

In the Stoic physics, body is "that which is spatially extended in three dimensions with resistance" (2.315, 319, 357, 381) which is the definition shared with Epicureans and Aristotle (e.g., *Phys.* 204b20). Place is what is occupied by a body, and void is defined as what can be occupied by a body but is not occupied (2.505; DL 7.140); void is also defined as "the absence of a body" (2.504), "an interval devoid of body" (SE 10.3). By definition, the void can accommodate fire produced by conflagration. The part of the void into which fire flows ceases to be a void and becomes the place. There arises, however, in this scheme one problem: how large the void is? The world is a sphere (2.547) and, as already claimed by Zeno, "there is no void within the world but an infinite [void] outside it" (1.95; DL 7.140). The void must be infinite, because if it were limited then there would be a problem with what is beyond the limit. If someone standing at this limit stretched out his hand, he would touch something. This could be neither a body, because all the bodies are inside the world, nor the void, because the extremity of the void has just been reached. This thought experiment (2.535) allows the Stoics to conclude that the void is infinite. However, all bodies are finite, so the void must not be a body: it is an incorporeal (2.503). As such, "it is intangible, and it has no shape and cannot be shaped, it is neither active nor passive, but only being capable of receiving a body"; moreover, it has "no up nor down nor before nor behind nor right nor left nor middle" (2.541, 2.557). As an incorporeal, the void does not affect the world in any way: it does not constitute any resistance and does not cause any change in the world. The world has nothing to do with the void because "only of the cosmos it can be said that it is self-supporting, because it alone has within itself all that it needs" (2.604).

And yet, the void is like a receptacle of matter; it is a precondition of the expansion during conflagration, so, were it not for the void, the conflagration could not take place. That is, the action of God on matter during this time would be adversely affected by the non-existence of the void. But by its nature, the void, as an incorporeal, does not exist; therefore, the Stoics resort to a terminological distinction between existence and subsistence. The void does not exist, but it does subsist, which is some imperfect kind of existence; that is, it does have and "must have some [sort of] reality" (2.541). The void is not a non-being; it is an incorporeal existent that becomes a condition for the possibility of conflagration. The void is not just a shadow of reality, it is an entity indispensable for the proper functioning of reality, at least, during conflagration.

During conflagration, fire into which the cosmos is transformed spills into the void, whereby the occupied part of the void becomes place: "according to its nature, it [the void] is infinite, but being filled, it is limited"; that is, it is not the void any more, but the place, because it is filled with a body (2.503). There is, thus, something in common between the void and the place: namely, they must be both three-dimensional (3D) entities to be occupied by bodies, but they are not resistant the way the bodies are. Except for being and not being occupied by a body, the difference between the void and the place is that the former is infinite and the latter finite. The place can thus be considered a finite part of an infinite void: it is finite because it is occupied by a body. The void is infinite because there is no body in it. The place and the void are interconnected and only their content indicates which is which, or as expressed by Hahm, place and void are coordinate species of a third thing, "that which is capable of being occupied by body."[4] Also, as already observed by Galen, the Stoics have to assume its three-dimensionality since the void is "in the nature of existing things" (2.502) which may be taken to mean that unless the void is 3D, a body – which by nature is 3D – would not be able to occupy it.

---

[1] Seneca, *Ep.* 58.13-15; SE 10.218; DL 7.135; A.A. Long, D.N. Sedley, *The Hellenistic philosophers*, Cambridge: Cambridge University Press 1987, v. 1, 163, 165, 301, but see also Jacques Brunschwig, *Papers in Hellenistic philosophy*, Cambridge: Cambridge University Press 1994, 97.

[2] Numbers after equal sign refer to Hans von Arnim, *Stoicorum veterum fragmenta*, v. 1-4, Leipzig: Teubner 1903-1924.

[3] Cf. Long, Sedley, *op. cit.*, 164, 181.

Three-dimensionality of the void is not contradicted by its having no shape[5]: infinity extended in three spatial dimensions, isotropic infinity in which no direction can be distinguished, could hardly be said to have a shape. It remains shapeless after a body moves in because the occupied part is not a part of the void any more – it is a place. Moreover, both time and void are said to be infinite. If infinity (plus infinite divisibility) is the only property of time and void, how could they be distinguished? The void has a spatial character, time temporal, and this character is reflected in the void in its inherent three-dimensionality.

The Stoics explicitly admit that there is one common attribute between place and body: they are both finite. There is, however, one more attribute: three-dimensionality. And because of the connection between the place and the void, three-dimensionality is also an attribute common to the void and a body. In this sense, the void can be considered a "possible attribute"[6] of a body. The void is a possibility of conflagration, a possibility of an action, and thus its infinity is a possibility of the finitude of the Stoic world. Finitude of the world is dependent on the infinity of the void. It is imaginable that the Stoic void exists by itself with no cosmos[7]; but the Stoic cosmos presupposes the existence of the void; that is, the finite presupposes the infinite. This dependence is reflected to some extent in the concept of the all which embraces both the finite cosmos and the infinite void.[8] The actuality of the void is the condition of the actuality of the world, and because the void is infinite, it is actual infinity, not potential infinity. This idea goes directly against Aristotle for whom only potential infinity was admissible. The Stoics implicitly disagree with this idea, thereby restoring the status of infinity that can be found in the Presocratics and in the Academy.

15.2 TIME

Zeno defines time as "the interval of movement which holds the measure and criterion of swiftness and slowness" (1.93). The definition is thus very similar to Aristotle's phrasing that time is the number of movement with respect to before and after (*Phys.* 218b1-2). To Aristotle, time is the number that is the measure of motion; to Zeno, it is an interval or extension of motion, and the length of the interval can also be expressed numerically. For both, time is a function of motion; that is, it depends on motion, motion determines time. Movement is possible without time, but time does not seem to be possible without movement. Time is instantiated when movement takes place, and because cosmos is in constant movement, there is always time alongside movement. Time can be analyzed independently of movement (for example, divisibility of time) which may give an impression that time is an independent entity. But this independence is only conceptual; it can only be "conceived of as existing by itself" (2.331).

Some independence of time is suggested in a definition proposed by Chrysippus. For him, time is "the interval accompanying the movement of the cosmos" (2.509). Time accompanies movement, follows it, is dragged by it, which may mean that it is an independent entity, but its behavior is determined by movement. However, such an interpretation seems to be rather unlikely. Time accompanies movement like a shadow accompanies a man. Without the man, the shadow ceases to exist, so not only its behavior but also its very existence hinges upon the existence of the man.

It is important to stress that time is a derivative term defined in terms of movement.[9] Also, Sambursky is right in stating that the cause-effect connection can be described as movement because it always involves a motion of the *pneuma*: "all causes are corporeal because they are portions of the *pneuma*" (2.340).[10] Only bodies act and only on bodies, but an effect of a cause can be only described by a verb. Thus an effect of a knife on a flesh is being cut (SE 9.341). This cause-effect connection is asymmetric which is reflected in the asymmetry of time, its "before" and "after."

---

[4] David E. Hahm, *The origins of Stoic cosmology*, [Cincinnati:] Ohio State University Press 1977, 105; Aristotle, on the other hand, "viewed void as a species of place, that is, a place that is temporarily devoid of body," so that there was no void outside the cosmos.

[5] As defended by Robert B. Todd, Cleomedes and the Stoic concept of the void, *Apeiron* 16 (1982), 129-136.

[6] Émile Bréhier, *La théorie des incorporels dans l'ancien Stoïcism*, Paris: Vrin 1928 [1908], 49; as he also says, the world requires "a theater without limits in which it by itself determines limits."

[7] The void is considered a "purely negative concept" in the sense that "it cannot be conceived only by elimination of the world and the body," as stated by Richard Goulet, Commentaries in Cleomedes, *Théorie élémentaire (De motu circulari corporum caelestium)*, Paris: Vrin 1980, 182, note 29, but the void still subsists even if we cannot grasp it.

[8] 2.522, 2.524; 3.Apoll. 9. Because Brunschwig sees "no link, either physical or conceptual, that can unite the body constituted by the world with the incorporeal constituted by the void," the all is to him merely an "ontological monster," Brunschwig, *op. cit.*, 98-99.

[9] On the one hand, Sambursky says that "the Stoics made nature prior to time and identified the flux of time with the passage of events in nature," on the other he maintains that "space and time are necessary conditions of every causal connection," Samuel Sambursky, *Physics of the Stoics*, New York: Macmillan 1959, 106, 52-53.

[10] Sambursky, *op. cit.*, 53. "The analysis of motion leads us eventually to what evokes it; the extension of movement and its present totality stem from the motive cause which is *causa perfecta et principalis*," Victor Goldschmidt, *Le système stoïcien et l'idée de temps*, Paris: Vrin 1953, 98.

The world is an ordered system of causal chains. Each event is followed by another event and "from everything that happens something else follows with a necessary causal dependence on it"; there is just no last event; the world will continue infinitely. But neither is there a first event, an event that has no cause.[11] An "uncaused event resembles a creation ex nihilo and thus is impossible." In this way, the world "is from infinity to infinity, self-evidently, and unceasingly" (2.945). The world is also structured: it is harmonious and perfect, it is divine, it is God himself. God is also fate, and fate is causality. Thus, the structure of the world has a causal character which is reflected in the derivative of this causality: time.[12]

In the Stoic doctrine, the status of time is not exactly analogous to the status of the void. The infinite void is required to exist independently of the Stoic universe for the universe to undergo the process of conflagration. There is no such requirement for time. Unlike the void, time is a shadow of the universe. Time is an Einsteinian entity, the void is a Newtonian entity. Both are incorporeals, but time is more so than the void. Both are infinite, but the void is infinite in its own right, time is infinite because God is capable of unceasing, continuous, and thus infinite activity. This activity is just possible by definition. There is no problem in the Stoic universe that this would require an infinite supply of energy that could be obtained from an infinite supply of matter converted to energy, which is impossible in a universe that is assumed to be finite since each body is finite.[13] As in the spatial aspect of the universe, so also in the case of time, infinity must be assumed, but infinity of time is a derivative of an infinity of recurrences of conflagration-palingenesia cycle which the Stoics simply state is happening without justifying it. One answer may be that eternal recurrences are "the only possible answer to the objection that to destroy (and remake) the world" is contrary to God's nature.[14] This would mean that even perfection of finite God requires infinity as its prerequisite. This, however, is not a general sentiment since the theory of eternal recurrences was rejected by some Stoics, to name Zeno of Tarsus, Diogenes of Babylon, Boethus of Sidon, and Panaetius.[15]

---

[11] It is not true, then, that in the cycles of eternal recurrences "there is no 'before' the cycle began and no 'after' the cycle ends" as claimed by John M. Rist, *Stoic philosophy*, Cambridge: University Press 1969, 282: "everything that happens, happens through antecedent causes," says Chrysippus (2.952), *ergo*, there is a "before" for the beginning of a new cycle.

[12] It is stated that "Chrysippus was unable or unwilling" to separate the concept of time from the concept of causal chain, Michael J. White, Time and determinism in the Hellenistic philosophical schools, *Archiv für Geschichte der Philosophie* 65 (1983), 51. Why should he be willing to do this if time is intended to depend on causality and to be understood in terms of causality?

[13] In the Eleatic spirit, we could say that because of the continuity of matter, matter is infinitely divisible, so a portion of matter is chosen and then a half of the portion is used as a supply of energy in one cycle, a half of the remaining half as a supply of energy in another cycle, etc. But this would not do, since all cycles are the same and thus require the same amount of energy, hence, infinite number of cycles require infinite energy.

[14] Jaap Mansfeld, Providence and the destruction of the universe in Early Stoic thought, in M.J. Vermaseren (ed.), *Studies in Hellenistic religions*, Leiden: Brill 1979, 169.

[15] 3.Zeno 5; 3.Diogenes 27, 3.Boethus 7; Cicero, *ND* 2.118.

For the Stoics, there is a global observer whose essence does not change, namely God. In fact, to the Stoics, God is more than just an observer of the cosmos, he is the cosmos. God, an active principle, is a body that is constantly active; therefore, time is a function of his motion. In the cycle of conflagrations and regenerations, when the end of one cycle is reached, there is a beginning of a new cycle in the sense that all occurrences are repeated in exactly the same way as the occurrences from the previous cycle, but they are different numerically. There will be Socrates living his life the same way as Socrates in the cycle before, but he is a different Socrates (2.625). They are identical copies of each other, but they are different because they live in different times. The difference between these times is guaranteed by God. He is an eternal, indestructible being, thus he continues his existence after conflagration, he does not begin it anew. The continuous existence of God and his actions guarantee that time also continues and does not repeat. To God, the two Socrateses are different just as two frames on two reels of the same movie which are displayed one after another are different. God can make a distinction between them by numbering them with two different integers (because there is no beginning of the world, there is no first integer in this counting sequence, so it may be assumed that God is counting with negative integers, too: …–123, –122, …). God is constantly active, he does not suspend his activity during conflagration,[16] so time can be represented with a continuous line punctuated with the conflagration periods and great year intervals.

The two cycles are the same in the sense that the same sequence of events occurs in both. But causality and motion do not stop at the end of a cycle. Conflagration is the cause of palingenesia, and, indirectly, each event is a cause of the same event in the next cycle. Although these two events look the same, there is one feature that distinguishes them, namely their position in the chain of events – the distance between these positions is measured by a time interval, according to the definition of time. "How to distinguish t, an instant in which occurs an event e_k, from t^*, an instant, at which occurs e_k^* [= a copy of e_k]?"[17] The answer is that there is an interval $[t, t^*]$ that separates the two instants and thus the instants are not the same. To the Stoics, the two instants are in a sense secondary to the interval itself whose endpoints they are. Between-conflagration worlds are thus distinguishable because they are carved out from different intervals in an infinite causal chain, which means they occur at different times. There is no reason, then, to "arrive at an absurdity: the infinite worlds of the cosmic cycle occur all at the same time; to sum up, there is only one world and there is only one world history."[18]

---

[16] It is said that at conflagration "there is no kind of regular movement; hence time is not even potentially measurable, and thus cannot exist," Rist, *op. cit.*, 281. This is only a conjecture that, like Plato, Chrysippus requires regular motions to measure time as probably suggested by the clause "the movement of the cosmos." But even if he did, we can consider God as endowed with a metronome made out of fire (so that it is not a subject of destruction during conflagration) located in his *hegemonikon*.

[17] Jonathan Barnes, *La doctrine du retour éternel*, in *Les stoïciens et leur logique*, Paris: Vrin 1978, 12.

[18] Barnes, *op. cit.*, 12.

Is circular time possible? Long suggests that such a possibility is "entirely appropriate" because the Stoics liken time to "the unwinding of the rope, bringing about nothing new and uncoiling what was primary" (2.944).[19] But this image fits linear time very well. To see in this image circular time, the rope should also be circular; that is, it should have the beginning tied to its end. The image conveys exactly what Ecclesiastes phrased about nothing new being under the Sun, and this hardly could be found as an expression of his belief in circular time. Today, the rope simile would be rephrased as the image of a film reel that is known to God from beginning to end because only God knows "the connection of all causes" and "would never be deceived," as Cicero says in a passage right before the rope simile.[20] Moreover, eternal recurrences are not understood as moving in a temporal circle and thus being the same. They are described as repetitious,[21] so, to the Stoics, one cycle would not suffice to speak of repetition. (In circular time there is no repetition; there is only one cycle.) Moreover, because a time line reflects the order of causes and effects, circular time would mean that the causal chain is circular; that is, each body is indirectly causing changes in itself through the chain of intermediate links. It seems, however, that such a possibility might not have occurred to the Stoics, and they would have found it an aberrant interpretation of the infinity of the causal chain. When it is said that to them "the management of the all is from infinity to infinity" (2.945), it would seem very unnatural and forced to view "from infinity to infinity" as an expression of moving around the causal circle. Had such an unorthodox possibility been meant, we could be quite certain that it would have been expressed much more explicitly. In any event, the possibility of a closed causal chain is theoretically admissible, but historically rather unlikely. Seeing circular time in conjunction with the Stoic doctrine is not really a hypothetical interpretation of the way the Stoics might have understood time but a view that is at best inspired by pronouncements of the Stoics.

With respect to the difference between parts of time, we have to keep in mind that time is defined as an *interval* of motion. An infinite past and infinite future are unions of such intervals and they just subsist. The present is also an interval, which may be very short or very long, and this leads to its peculiar ontological status. Only the now is real because the world is real and its motion is real. However, the reality of the now is fleeting. There is no "minimal time," no "partless now" (2.519). The now is not an instant, not a point, not an Aristotelian "limit of

---

[19] A.A. Long, The Stoics on world-conflagration and everlasting recurrence, *Southern Journal of Philosophy* 23 (1985), supplement, 29, 33 note 37.

[20] The rope simile "simply expresses the idea that the things [in the world] are present together (*ensemble*): the temporal succession only deploys the collection (*ensemble*) given at the beginning," where the collection is the totality of one cosmic period, Goldschmidt, *op. cit.*, 80.

[21] "The world again afresh (πάλιν ἐξ ὑπαρχῆς) is restored to the same [state as before]" (2.625); Logos creates again the same world and "such periods return from eternity without ceasing" (2.599); the Stoics say that "the same I is born again in παλιγγενεσία" (2.627). The repetitious character of eternal recurrences is built into the name of the recreating phase, *palingenesia*.

time."[22] The now is always an interval and regardless of its length, the now is never present in its entirety because "no time is wholly present, but only broadly speaking" (2.509); it is not presence *sensu stricto* as in the case of bodies, but *sensu largo*, speaking inexactly, as the reflection of the existence of the motion of bodies. The now is an interval consisting of the past and the future (2.519), or as expressed by Archedemus, the "now is a kind of joining and meeting of the past and future" (3.Arch 14).[23] This interval is a meeting section of time, a juncture that binds together past and future to render continuous, infinite time.[24] Infinity is imperfectly present in past and future: both are infinite in one direction only, whereas time is infinite "in every respect," as later phrased by Posidonius (Aetius 1.8.42). The past, the future, and time in general are 1D anisotropic infinities, where their anisotropy reflects that of cause-effect relation. If the present is not infinite and not an interval, it ceases to have any temporal characteristics and thus ceases to be time. To retain its temporal character, it simply has to be an interval, not a point.[25]

15.3 CONTINUUM

According to the Stoic physical doctrine, the world and all things in it are the result of an active principle, God-*pneuma*, acting on the passive principle, matter. The two principles are never found in separation; *pneuma* pervades matter in all its parts which results in a complete mixture of the two principles. This interpenetration is also prevalent on a smaller scale. Soul (which, after all, is an offshoot of the cosmic *pneuma*) pervades the human body, fire pervades heated-up iron, and poison takes its effect by being blended in the body. In such a mixture, one body is diffused in another, but both bodies preserve their properties and they are mutually coextended, and "there is no part that does not participate in all the [bodies] in such a mixture" (2.473). This view of the interpenetration of the bodies assumes that the bodies are continuous entities. If they were composed of atoms, interpenetration would stop on the atomic level since atoms, by their nature, would not interpenetrate.

---

[22] *Phys.* 222a10. Importantly, to Aristotle, "in so far as the now is a boundary, it is not time, but an attribute of it," *Phys.* 220a21.

[23] Archedemus' description hardly indicates that he "took over his [Aristotle's] description of 'the now' wholesale," A.A. Long, D.N. Sedley, *The Hellenistic philosophers*, 307.

[24] "The Stoic present" is "an infinite sequence of nested time intervals shrinking towards the mathematical 'now'," "a shrinking duration of only indistinctly defined boundaries," Sambursky, *op. cit.*, 104, 105.

[25] White's reference to infinitesimals and his interpretation of the now as an indefinite infinitesimal interval does not shed any new light on the interpretation of the now as a plain interval, Michael J. White, Zeno's arrow, divisible infinitesimals, and Chrysippus, *Phronesis* 27 (1982), 247. In fact, he himself later shows at some length how inadequate is the use of infinitesimals to interpret Chrysippus' views, Michael J. White, *The continuous and the discrete*, Oxford: Clarendon Press 1992, 311-314.

A cardinal feature of a continuum is its divisibility. For Chrysippus, just as for Aristotle, a continuum is divisible; however, there is no point at which divisibility can stop: a continuum can be divided indefinitely because the result of a division is also a continuum and thus it is also divisible. "Division (τομή) is to infinity [says Apollodorus]; it is [rather] infinite, says Chrysippus; for there is not some infinity into which the division reaches, but it is endless" (DL 7.150). Chrysippus seems thus to state that division cannot be concluded by reaching an elemental level of division; there are no indivisibles, no atoms, and each division is just a stepping stone to another division. When begun, division cannot be stopped; it is an inherently endless, infinite process. Infinite division would be concluded after some infinity of indivisibles were reached, but by the nature of the Stoic continuum, there are no such indivisibles. Actual infinity of parts or "division to infinity" does not exist because there are no parts, not because infinity can at best be only potential, as Aristotle taught. From today's perspective, by the way, the argument it not entirely inadmissible. If infinite divisibility is only denumerable, then division of, say, an interval cannot reach the level of points, unless extracting endpoints of a closed subinterval is also considered division; but divisibility seems to be understood by Chrysippus in its literal sense as cutting (τομή) a continuum into two subcontinua.

Division cannot reach indivisibles, because there are none, but can a body be divided into any parts? It depends on the level of analysis, says Chrysippus. If we look at the problem of parts "with regard to the inexact question we will reply that we consist of head, trunk and limbs," but when asked about ultimate parts, we "must say neither of which [parts] we consist, nor, likewise, of how many, either infinite or finite" (2.483). On the phenomenal level, we can distinguish some parts of a body, but we must realize that these are not ultimate parts; they can also be subjected to division. The parts we distinguish are largely conventional and can always be changed. For the Stoics, the whole and the part are in our minds, whereas objects are neither whole nor part (2.80). There is nothing naturally fixed in the parts themselves; a body can always be divided differently, one reason being that the borderlines between bodies can be difficult to delineate. Where does the head end and the neck begin? Moreover, the concept of the borderline between parts is of a different category than the parts themselves because the Stoics viewed the limits of the bodies to subsist only in the mind (2.488). "It is all in the mind" makes the Stoics sound very much like Kant.[26] Because distinguishing a particular set of parts in a body is ontologically noncommittal, so is not the cardinality of this set: it can be finite, it can be infinite.

---

[26] Watson views the Stoics as forerunners of Kant; as he says, "*lekta* are the patterns which the mind tends naturally to impose on reality," Gerard Watson, *The Stoic theory of knowledge*, Belfast: Queen's University 1966, 28. "'Things' as we grasp them are not 'natural', but are products of human intellect and its abstraction, a result of articulation," Jan Pinborg, Das Sprachdenken der Stoa und Augustins Dialektik, *Classica et Mediaevalia* 23 (1962), 156; similarly Johnny Christensen, *An essay on the unity of Stoic philosophy*, Copenhague: Munksgaard 1962, 51. Because of the continuity of the cosmos, slices of cosmos are not fixed, Andreas Graeser, *Zenon von Kition. Positionen und Probleme*, Berlin: de Gruyter 1975, 19.

The possibility of endless division allows Chrysippus to give a main characteristic of an infinite set: an infinite set has a proper subset of the same cardinality as the set itself.[27] Plutarch says that one must be a Stoic not to see as self-evident that "man consists of more parts than man's finger, and the world than man" (Plutarch, *CN* 1079a = 2.484); that is, to the Stoic, one can keep on dividing endlessly man's finger just as man himself can be a subject of infinite division; because "division pulverizes (βράττει) bodies to infinity and among infinities there is no greater or smaller," continues Plutarch, man's finger can be pulverized to the same extent as man's entire body; because all infinities are of the same cardinality (presumably, denumerable), man's finger can be pulverized into the same infinite number of parts as the body of which it is already a part.

15.4 *LEKTA*

According to the Stoics, there are three things involved in understanding an utterance: the signifier, that is, the utterance itself, for example the word "stone"; the object, that is, the stone indicated by the word; and the thing signified, or the *lekton* (SE 8.11-12). "*Lekton*" is a neuter adjective of "*legein*," and "*legein*," "to say," is defined as "to utter the sound capable of signifying the object conceived" (SE 8.80), where the object conceived is apparently the *lekton*.[28]

The need for the third entity – next to the utterance and the object – arises in the situation when a Greek and a barbarian hear the same word; the barbarian does not understand the word although he hears the same sound as the Greek. The *lekton* is induced by the word in the mind of the Greek but not in the barbarian's mind. The Greek and the barbarian have the same representation in their minds, the same intelligible object, but to the former a connection is made between the word and the object which is lacking to the latter. This connection between the stone and the word "stone" is an attribute of the stone of being signified by "stone." However, this attribute should not change the nature of the object, and so it must be an incorporeal. This is a departure from Aristotle, to whom, in the words of his commentator, Ammonius, utterances signify thoughts and objects through thoughts as intermediaries. But the Stoics include the *lekton* as an intermediary between the thought and the object (2.168). By introducing *lekta*, the Stoics avoid the problem of determining how the thoughts – modifications of the commanding faculty (ἡγεμονικόν), and thus a form of an expression – of two people, or, for that matter, the thoughts of the same person at different moments, can have the same meaning. For the Stoics, comparison of thoughts is possible because there is a *tertium comparationis*, namely the objective meaning of the thought, the *lekton*. Thinking is a subjective process of independent minds and as such, it cannot be transferred from one mind to another to make communication possible. But by introducing an invariant, the *lekton*, the Stoics enable this communication.

---

[27] Sambursky is right to point to this implicit definition given by Chrysippus, *op. cit.*, 97; cf. Josiah B. Gould, *The philosophy of Chrysippus*, Leiden: Brill 1970, 117.

[28] Adam Drozdek, *Lekton*: Stoic logic and ontology, *Acta Antiqua Academiae Scientiarum Hungaricae* 42 (2002), 93-104.

Lekton is defined as "that which subsists according to a rational presentation" (SE 8.70; DL 7.63). So the *lekta* are not independent entities. The presentation implies the existence of the mind on which it is imprinted; therefore, "just as 'walking' entails the existence of a walker, so *lekta* would appear to entail the existence of a mind."[29] However, this does not mean that the *lekta* are constructs of the mind. They do not exist independently of the mind, but, at the same time, the mind does not create them.[30] The definition of the *lekton* also suggests that it exists as long as the expressions corresponding to it.[31] The mind, however, invokes a *lekton*; it does not construct it. The mind has no part in shaping the *lekta*, otherwise, the objectivity of the *lekta* would be destroyed.

The world of *lekta* is primarily a semantic entity, the world of meaning, but it is not structure-free. It has its syntax, so the *lekta* are semantico-syntactic entities.[32] Their role is to be invariants behind actual utterances in whatever language and at whatever time they can be phrased. It is interesting to see an analogy between this understanding of the *lekta* and an effort to construct an interlingua as a tool for automated translation of languages.

A great challenge in machine translation is to minimize the amount of work needed in translating many languages into many other languages. For n languages, there are needed $n(n\text{-}1)$ translation systems to translate languages into one another. It is possible, however, to translate each language into one common language, an interlingua, and then intelingua into each language, which requires only $2n$ translation systems. From a purely practical point of view, the interlingual approach is more appealing than the language-to-language approach for at least four languages, $n > 3$.

In the syntactic interlingual approach, interlingua turns into summalingua that includes syntactic categories of all languages. In the semantic approach, interlingua is envisioned as a web of meanings lying behind all languages. One interesting approach is the conceptual dependency theory proposed by Roger Schank. The main assumption of the theory is the statement that "the basis of natural language is conceptual. That is … there exists a conceptual base that is interlingual, onto which linguistic structures in a given language map during the understanding process and out of which such structures are created during generation."[33] Schank proposes an intricate structure of the interlingua. It consists of different types of

---

[29] A.A. Long, Language and thought in Stoicism, in A.A. Long (ed.), *Problems in Stoicism*, London: The Athlone Press 1971, 90, 94; also Nuchelmans seems to lean in this direction, Gabriel Nuchelmans, *Theories of the proposition*, Amsterdam: North-Holland 1973, 86-87.

[30] *Lekta* are mental constructs according to Gerard Watson, *op. cit.*, 42; Christensen, *op. cit.*, 25; Graeser, *op. cit.*, 19.

[31] Long, Language and thought, 97; Andreas Graeser, The Stoic theory of meaning, in J.M. Rist (ed.), *The Stoics*, Berkeley: University of California Press 1978, 88-89.

[32] This is also phrased as that the *lekta* "have a syntax as well as a model theory," Urs Egli, Stoic syntax and semantics, in *Les stoïciens et leur logique*, 138.

[33] Roger Schank, Conceptual dependency: a theory of natural language understanding, *Cognitive Psychology* 3 (1972), 553-554.

concepts, conceptual cases, and dependencies, which are relations between all these conceptual categories. He gives a set a rules specifying permissible conceptual dependencies, conceptual syntax rules, or syntax of the conceptual level, which forms the grammar of meaningful semantic relationships. It is important that interlingua is a conceptual language, a language of concepts, that is, a semantic entity that has its syntax. Although semantics is a more important side of interlingua, the interlingua is a semantico-syntactic union. It seems that the Stoic *lekta* were intended to play the role of an interlingua.

There remains a problem of the eternal subsistence of the *lekta* and their objectivity to invariability of meaning. In the Stoic system, the dependent subsistence of *lekton* does not necessarily mean that they cannot be eternal. It may be assumed that the Stoic Logos-God is capable of holding all possible statements expressing all possible *lekta* in his mind, in his divine *hegemonikon*, and thus all the corresponding *lekta* timelessly subsist.[34] Therefore, anyone who utters a particular expression refers to the same *lekton* as the expression existing from eternity in God's mind. Importantly, truth value of the *lekton* may evolve. The truth value of the *axioma* (a particular type of the *lekton*) corresponding to "Dion is dead" today may be different from that of the *axioma* corresponding to this expression tomorrow, after Dion's death, but, still it is only one *axioma* at any one time.

It is important to notice that there is no fundamental impossibility that God thinks at the same time all thoughts that induce the subsistence of requisite *lekta*. The universe is finite and thus God who is immanent in the universe is also finite. But although the world exists eternally, it undergoes periodic conflagration after which the same events happen in exactly the same order. Therefore, the number of different people and different thoughts and utterances is also finite. Incidentally, even if the Stoic doctrine did not include the thesis of eternal recurrences, God could accommodate an infinite number of the *lekta* as well. The Stoics could refer to Zeno's dichotomy paradox and say that some unit of the *hegemonikon* corresponds to all the thought sentences: a half of this unit corresponds to one sentence, one fourth to the second sentence, one eighth to the third sentence, etc. In this way, even a finite God can be a guarantor of the existence of an infinity of sentences and thereby of the subsistence of an infinity of *lekta*.

God-Logos saturates entire reality. He is present in all the nations, in all rational beings, regardless of what language they use, and thereby he can be a guarantor of the timeless subsistence of the *lekta*, of the eternal presence of the meanings of spoken and thought, or yet to be spoken and thought, statements. By the nature of the universe, there is one underlying interlingua, an eternal repository of meanings to which all languages of all rational beings can connect. The interlingua would thus be an incorporeal counterpart of the Stoic *cosmopolis* that transcends all institutions.

---

[34] Kahn is convinced that "there should be some connection between the *lekta*" and the cosmic Logos, although the only suggestion he offers is that the *lekta* are "the mode of rationality reflected in human speech" as reflected in the word "logos," Charles H. Kahn, Stoic logic and Stoic Logos, *Archiv für Geschichte der Philosophie* 51 (1969), 169.

15.5 THE FINITE AND THE INFINITE

The Stoic world is finite in the sense of limitedness. The hierarchy of beings in the world is also limited (cf. Cleanthes' proof of the existence of God from the hierarchy of perfection, 1.529). Ontologically, the world is immersed in infinity. The body-finger definition recalled by Plutarch is general enough to include infinities on any level: the infinitely small and the infinitely large. A less general definition given by Chrysippus which states that "the infinite has neither the beginning, nor the middle, nor the end" (*SR* 1054b = 2.539) does not allow for any limits; so, a closed interval would not be infinite. But in his definition, as the context indicates (the discussion of the void), Chrysippus means the infinitely large. The two definitions of infinity are givens to the Stoics, they are non-problematic, taken for granted. Therefore, the Stoics have little problem in immersing a finite world in infinity.

The world is surrounded by infinite void; it has infinite duration, and it is a continuum, thus it is infinitely divisible. Epistemologically, this finite world can be understood if infinity is grasped. In the Stoic doctrine, the finite world is infinite because almost everything in it is a continuum: "bodies are divided to infinity, and similarly things comparable to bodies, such as surface, line, place, void, and time" (2.482, the *lekta* and *ennoemata* are excluded).[35] Body and place are 3D anisotropic limited continua, but a body has also solidity; the void is a 3D isotropic unlimited continuum; time is a 1D anisotropic continuum (limited or unlimited); place, void, and time are incorporeals. To understand the continuous character of the world, one has to know what a continuum is and this requires knowing the concept of infinity. With infinity, a concept of relation can be developed and then the concept of an ordering relation which is needed to understand an order in a set, such as the set of events that form causal chains.[36] Nothing changed today: to define continuum in an axiomatic set theory, the axiom of infinity has to be included among the premises, which, basically, states that there exists an infinite set. Infinity has to be thus assumed at the very outset even to grapple with the finite. The finite world is built upon and can be understood through infinity.

---

[35] Bloos says that it is unclear to him why the void is included among infinitely divisible entities because he considers it to have no dimension "and no properties at all can be predicated about it, so it is meaningless to say that it is infinitely divisible," Lutz Bloos, *Probleme der stoischen Physik*, Hamburg: Buske 1973, 81. But Bloos sees no problem with the divisibility property being ascribed to other incorporeals, place and time. Moreover, it is dubious, as Bloos assumes without any discussion, that the claim of the void having no dimension can be defended. Also, the void's three-dimensionality and divisibility indicate that it is much more than "pure indetermination," as claimed by Tony Lévy, *Figures de l'infini*, Paris: Seuil 1986, 63.

[36] White is correct in saying that the concept of "the sum of cosmic motion as an eternally fixed, linear series of some sort" is needed to define time, which can be accomplished by imposing "a linear ordering on the sum of these states" and he suggests, in effect, that a set of equivalence classes with respect to the relation of simultaneity defines time, Time and determinism, 50.

---

16 INFINITY IN GREEK MATHEMATICS

Infinity is one of the most important aspects of mathematical structures, but analysis of infinity itself came rather late in mathematics. In Greek mathematics, infinity never became an object of mathematical analysis. It was taken for granted from its very inception, beginning with Greek arithmetic, i.e., a theory of natural numbers, the only numbers recognized by the Greeks.

16.1 NONNEGATIVE INTEGERS

Book 5 of Euclid's *Elements* presents a general theory of proportion that can be applied in geometry, arithmetic, and music, with, as it were, geometrical bias since magnitudes are represented by straight lines. The theory is attributed to Eudoxus and can be considered a response to the problems raised by the discovery of incommensurables.[1] The Pythagoreans also had a theory of proportions, but the theory was limited to numbers, although it was very likely also applied in geometry, probably only to commensurables. If so, proofs based on proportion would be very limited in scope if incommensurability were not taken into account. It is rather striking that the discussion of the general theory of proportion of magnitudes in book 5 is followed by a discussion of a specific theory of proportion of nonnegative integers in book 7. Book 7 repeats to a large extent the results of book 5, applying them to numbers as a particular kind of magnitudes and only in one place (prop. 10.5) does Euclid point to the connection between the two chapters. This redundancy can be explained by the fact that Euclid simply presented a traditional theory of proportion before Eudoxus gave it a general form.[2] This traditional arithmetic and theory of numerical proportions is given in books 7 through 9.

A traditional rendering of numbers in book 7 is clear from the very inception when a unit (a monad) is defined somewhat cryptically as "that because of which each of the beings is called one" (def. 1) and a number as "a multitude composed of units" (def. 2). That is, a unit is not a number, which, when strictly followed, requires having different proofs for the unit and for numbers, and Euclid sometimes does just that, but sometimes he ignores the difference.

---

[1] Thomas Heath, *Thirteen books of Euclid's Elements*, Cambridge University Press 1956, v. 2, 112.

[2] Heath, *op. cit.*, v. 2, 113. "It is my judgment that this entire book should be attributed to the Pythagoreans before Archytas," says B[artel] L. van der Waerden, *Science awakening*, New York: Oxford University Press 1961 [1950], 110; "book vii was a textbook on the elements of the theory of numbers in use in the Pythagorean school," p. 115. It is not possible to decide: bk. 5 could have been written before bk. 7, after it, or at the same time, even by the same author, argues Jean Itard, *Les livres arithmétiques d'Euclide*, Paris: Hermann 1961, 67-68.

There is no limit imposed on how many integers there are, and thus an infinite number of them can be supposed to be an obvious assumption for mathematicians.[3] However, if for each number n it is possible to construct a number greater than n, there has to be an infinite number of units to construct larger and larger sets. Also, because the sets can be constructed from different units, there are an infinite number of 2s, an infinite number of 3s, etc. A possibility of constructing larger and larger numbers is explicitly given as prop. 9.20 which states that there are more primes than any supposed number of primes; that is, there is an infinite number of prime numbers. The theorem is proven by showing that, for the first primes $p_1, p_2, \ldots, p_n$, a prime p that divides the number $p_1 \cdot p_2 \cdot \ldots \cdot p_n + 1$ must be greater than p_n, i.e., p is in the interval $(p_n, p_1 \cdot p_2 \cdot \ldots \cdot p_n + 1]$.

An infinity of numbers is assumed in many theorems, particularly when they open with the statement that says "if there are as many numbers as we please." Proofs of such theorems, however, are made by example rather than for a general case and it is left to the reader to investigate the proof for other cases. As a typical example, consider prop. 7.12 which states:[4] "if there be as many numbers as we please in proportion, then as one of the antecedents is to one of the consequents, so are all the antecedents to all the consequents," which can be rephrased as: for any number of integers $n_1, \ldots, n_i$ and $m_1, \ldots, m_i$,

$$\text{if } n_1 : m_1 = n_2 : m_2 = \ldots = n_i : m_i, \text{ then } (n_1 + \ldots + n_i) : (m_1 + \ldots + m_i).$$

In his proof, Euclid says, "let A, B, C, D be as many numbers as we please in proportion," i.e., he proceeds for the case of four numbers only and then states, if A : B = C : D, then (A + C) = (B + D). In a situation like this, we would use today the proof by mathematical induction, but the principle was first formulated by Pascal. However, some adumbration of the principle can be found in Euclid. In prop. 7.2 he gives a method to find the greatest common measure for two numbers that are not relative primes, the method which is still used today to find the greatest common divisor $\gcd(k,l)$. Assuming that $l \geq k$, we execute the loop to perform successive subtractions (ἀνταναίρεσις or ἀνθυφαίρεσις)[5] by subtracting a smaller number from the larger until the numbers become equal, and the obtained value is the greatest common divisor:

```
gcd($k,l$)
  while $k \neq l$
    $d = l - k$;
    $l = \max(k,d)$;
    $k = \min(k,d)$;
  return $k$;
```

---

[3] Ian Mueller, *Philosophy of mathematics and deductive structure in Euclid's* Elements, Cambridge: MIT Press 1981, 60.

[4] Other proofs of this type are given in propositions 7.1-6, 8-10, 14, 20-22, 27, 33, 36; 8.1, 2, 4, 6, 7, 9, 10, 13; 9.8-14, 17, 20-23, 35, 36, i.e., ca. one third of all proofs in arithmetic books 7 through 9; Itard, *op. cit.*, 73.

[5] See Euclid's def. 5.3 and Heath, *op. cit.*, v. 2, 120-121; Oskar Becker, *Das mathematische Denken in Antike*, Göttingen: Vandenhoeck & Ruprecht 1966 [1957], 103; Itard, *op. cit.*, 25-32.

Then, basing on this method, he shows in prop. 7.3 how to find the greatest common measure for three numbers, $\gcd(k,l,m) = \gcd(\gcd(k,l),m)$. Apparently, he considered it to be a sufficient indication that, when needed, we could find the greatest common measure by extending this method to $\gcd(k,l,m,n) = \gcd(\gcd(k,l,m),n) = \gcd(\gcd(\gcd(k,l),m),n)$, etc. so that, eventually, if gcd is found for i integers, it can also be found for i+1 integers. In fact, he did implicitly consider it since in the proof of prop. 7.33 he required that the greatest common measure should be found for any number of integers.[6] Similarly for prop. 7.27 which only states that for relative primes A and B, numbers A^2 and B^2 are also relative primes and so are A^3 and B^3. However, the proposition is implicitly used in general form (for relative primes A and B, for any n, A^n and B^n are also relative primes) in the proof of prop. 8.2.[7]

The infinite descent method favored by Fermat is found in the proof of prop. 7.31. The method is a combination of reversed mathematical induction and proof by contradiction: if it can be shown that when n has a property P, then any $m < n$ has this property, then it can be concluded that the property is possessed by an infinity of numbers less than n, which, of course, is impossible for natural numbers.[8] Euclid's prop. 7.31 states that each nonprime is divisible by a prime. If B is a nonprime divisor or a nonprime A, then a divisor C of B is found which would also be a divisor of A. If C is a nonprime, a divisor D of C is found which would also divide A. The process continues until a prime divisor is found, otherwise an "infinite [series of] numbers will measure [= divide] the number A, each of which is less than the other, which is impossible in numbers." Numbers, i.e., positive integers, have no upper limit, but they do have a lower limit, which is the unit.

16.2 INCOMMENSURABILITY

Natural numbers were the only numbers recognized by Greek mathematicians. Fractions were not really numbers; they were handled in terms of proportions between natural numbers. Quite early mathematicians learned that natural numbers are insufficient to express all the relations between various magnitudes, in particular between various lengths.

---

[6] Heath, *op. cit.*, v. 2, 302-303; Mueller, *op. cit.*, 68-69, 79; Itard, *op. cit.*, 73, 88.

[7] There is even an apparently spurious addition in the phrasing of 7.27 that explicitly states the validity of the theorem for any powers A^n and B^n of relative primes A and B, Heath, *op. cit.*, v. 2, 328-329, 347; Itard, *op. cit.*, 115, 131.

[8] It is sometimes posited that prop. 7.31 uses the least number principle, according to which there is always a smallest number in a nonempty set of natural numbers, Mueller, *op. cit.*, 77-78. Although the infinite descent method and the least number principle are often equated (e.g., Edward Nelson, *Predicative arithmetic*, Princeton: Princeton University Press 1986, 29), the identification does not go unchallenged, cf. Yvon Gauthier, Logical and philosophical foundations of arithmetical logic, in A.D. Irvine, *Physicalism in mathematics*, Dordrecht: Kluwer 1990, 332-333. For nonidentity of infinite descent and mathematical induction, see Morris Kline, *Mathematical thought from ancient to modern times*, New York: Oxford University Press 1990 [1972], v. 1, 275.

The discovery of incommensurability is ascribed to Pythagoras himself. A very late testimony of this ascription comes from Proclus (*In Eucl.* 65.16-20). However, it is very likely that Proclus or his source (possibly Eudemus) wrote not about Pythagoras' discovery of the doctrine of incommensurables (τὴν τῶν ἀλόγων πραγματείαν), but about the doctrine of proportionals (ἀναλόγων) or proportions (ἀναλογιῶν).[9] The Pythagoreans investigated the problem of incommensurability. For instance, Theodorus of Cyrene who is said to be a Pythagorean (Iamblichus, *Vita Pyth.* 267) proved the irrationality of $\sqrt{3}, \ldots, \sqrt{17}$ (Plato, *Theaet.* 147d). The sequence does not include $\sqrt{2}$ which may mean that the proof of its irrationality was known before Theodorus. Very likely, the first proof of incommensurability of the diagonal of the square was performed by contradiction because the proof led to the conclusion that odd numbers would be even (Aristotle, *An. pr.* 41a23-27, 50a35-38),[10] and such a proof is provided in the interpolated prop. 10.117.

Assume that the side s and diagonal d of a square are commensurable and that $d : s = k : m$ for k and m being the smallest numbers, i.e., they cannot be both even numbers. We have $d^2 : s^2 = k^2 : m^2$, but by prop. 1.47 (the Pythagorean theorem), $d^2 = 2s^2$ so that $k^2 = 2m^2$, whereby k^2 and thus k are even so that m is odd. Suppose that $k = 2j$, thus $k^2 = 4j^2 = 2m^2$, whereby $m^2 = 2j^2$ and so m^2 and thus m are even – a contradiction with the fact that m is odd.

The discovery of incommensurability indicated that the science of numbers and numerical proportions has its limitations, but this only gave an impulse for the further development of mathematics. The Greeks did not rework the concept of the number, but tamed incommensurability by generalizing the theory of proportions by investigating proportions of magnitudes, not just proportions of numbers. Magnitudes were illustrated geometrically as line intervals and in that way addressing the problem of incommensurability became a major impetus for the development of geometry. As mentioned in the previous section, such a general theory of proportions is presented in book 5 of Euclid's *Elements*. A general approach to magnitudes that overcame the limitations of arithmetic of natural numbers by encompassing irrational magnitudes allowed mathematicians to address many difficult problems, one of which was the problem of squaring the circle.

16.3 QUADRATURE OF THE CIRCLE

An attempted quadrature of the circle is ascribed to Anaxagoras while he was in prison (59A37). The popularity of the problem at the end of the fifth century is demonstrated by Aristophanes' mocking mention of an astronomer who "with the

---

[9] G[ustav] Junge, Wann haben die Griechen das Irrationale entdeckt?, in *Novae symbolae joachimicae*, Halle/S.: Weisenhaus 1907, 261.

[10] Aristotle uses the example of incommensurability of the side of a square and its diagonal 26 times in his writings, not once attributing the discovery to the Pythagoreans, Junge, *op. cit.*, 239-240.

straight ruler set to work so that the circle becomes four-cornered (τετράγωνος)" (*Birds* 1004-1005).[11]

An early attempt at squaring the circle was undertaken by Antiphon, probably the sophist who disputed with Socrates (Xenophon, *Mem.* 1.6.1, DL 2.46), and his attempt is described in commentaries on Aristotle's *Physics* (Themistius, *In Phys.* 4.2-8; Simplicius, *In Phys.* 54.20-55.24; Philoponus, *In Phys.* 31.9-32.3). Antiphon tried to exhaust (δαπανᾶν, Simplicius, *In Phys.* 55.6, 20) a circle by starting with a square (Simplicius, or an equilateral triangle (Themistius)) inscribed regular polygons in the circle by doubling the number of sides in each step. In this way, he obtained a better and better approximation of the circle which, when continued indefinitely, led to the exhaustion of the circle: Antiphon "thought that in this way the area [of the circle] would be exhausted and we should some time have a polygon inscribed in the circle the sides of which, because of their minuteness, would coincide with the circumference of the circle. And since we can make a square equal to any polygon … we shall be able to make a square equal to a circle" (Eudemus, *ap.* Simplicius, 55.6-11). As observed by Alexander, geometrical principles are violated in this conclusion since a circle can touch a line in one point only. Simplicius agrees with this observation and states that "by continually bisecting the space between the chord and the arc, it will never be exhausted, nor shall we ever reach the circumference of the circle even though the cutting should continue *ad infinitum*" (55.19-21). Antiphon, by his conclusion – states Themistius – effectively "abolished infinite division which the [true] geometer assumes as his foundation." The conclusion that a circular polygon can be generated if its sides are sufficiently small may have been inspired by Protagoras who argued with geometers by saying that a line touches a circle at more than one point (Aristotle, *Met.* 998a2-4).

The method proposed by Antiphon was finitistic by claiming that the bisection of inscribed polygons eventually results in a circle. A polygon of infinite number of sides is hardly meant here, as is rather clear from Simplicius' report and his comment on it. Antiphon did not think infinitistically, and if he did, he was sidetracked by the ponderous character of empirical lines, i.e., by not thinking of lines that have no width. However, he opened the road for an expansion of his approach into the exhaustion method. Incidentally, the original (finitistic) method of Antiphon was actually used by Archimedes in his *Measurement of the circle* to find the lower limit for π, i.e., $\pi > 3\frac{10}{71}$, found by determining the perimeter of the 96-sided inscribed polygon.

Antiphon's approach was apparently soon extended by Bryson who used not only inscribed but also circumscribed polygons (Alexander, *In Soph. elench.* 90.10-21, Themistius, *In An. post.* 19.11-20, Philoponus, *In An. post.* 111.17-114.17). However, the details of Bryson's approach are lacking. Apparently, after inscribing and circumscribing squares (Alexander, or any polygons) in and about a circle he just concluded that an intermediate square (later conjectured to be an arithmetic mean of the two squares or maybe their geometric mean) is equal to the circle because “things greater and smaller than the same thing are simply equal” (Alexander 90.16-17).

---

[11] Τετράγωνος is really a pun on the method of squaring (τετραγωνισμός) the circle; the astronomer in the *Birds* actually draws two lines perpendicular to one another that cross at the center of a circle representing a city, i.e., he draws a four-corner (a cross) at the center, not a square.

There was a doubt about whether the argument should be taken seriously as being too simplistic.[12] However, because of the principle that “things greater and smaller than the same thing are simply equal,” Bryson is considered to have foreshadowed Dedekind’s method of cuts.[13] This conjecture can be reinforced by the report that Bryson saw the circle to be greater than all inscribed polygons and smaller than all circumscribed polygons, which may suggest that he did generate a sequence of inscribed polygons with an increasing number of sides and areas, like Antiphon, and series of circumscribed polygons with an increasing number of sides and decreasing areas. This method certainly would explain why Bryson’s name is repeatedly mentioned by Aristotle and his commentators.

The approach of Antiphon and Bryson culminates in the method of exhaustion that is attributed to Eudoxus (Archimedes, prefaces to *On the sphere and cylinder* i and to *Quadrature of the parabola*). The basic idea of the method is to approximate the area of a curved figure by a successive generation of polygons which cover more and more area of the curve in an attempt to cover it, or exhaust, completely. A similar procedure can also be used for solids to approximate their volumes. This method is used exhaustively in book 12 of the *Elements*.

The method of exhaustion presented in Euclid’s *Elements* is based on prop. 10.1 which states that for two unequal intervals, when from the longer interval more than a half of its length is cut off, and from what is left, more than a half of its length is cut off, etc., then eventually the remaining part of the original longer interval will be shorter than the original shorter interval.

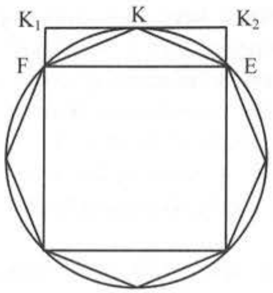


Figure 1

---

[12] With his argument “Bryson hardly deserved any place in the history of mathematics,” Johan L. Heiberg, Griechische und römische Mathematik, *Philologus* 43 (1884), 336, because the argument is but a “clumsy sophism,” Ferdinand Rudio, *Der Bericht des Simplicius über die Quadraturen des Antiphon und des Hippokrates*, Leipzig: Teubner 1907, 109.

[13] Becker, *op. cit.*, 94.

Prop. 12.2 states that areas A_1 and A_2 of two circles are to one another as the squares of their diameters d_1 and d_2, i.e., $A_1 : A_2 = d_1^2 : d_2^2$. First, Euclides shows that a circle can be exhausted by polygons. An area of a square inscribed in a circle of area A is more than a half of A since the area of this square is exactly a half of the area of a circumscribed square which, of course, is larger than A. Then each arc FE corresponding to a side FE of the square is bisected and an octagon is formed by drawing lines FK and KE for each side of the square (Fig. 1). Let us observe that the area of the rectangle FK_1K_2E is obviously greater than the area of the circle segment FKE, and because the area of triangle FKE is exactly a half of the area of the rectangle FK_1K_2E, the area of this triangle is greater than a half of the area of the circle segment FKE. Therefore, the octagon includes the square and over a half of the difference between the area of the circle and the area of the square. The process continues by forming a 16-sided polygon, a 32-sided polygon, etc., each polygon including the area of the previous polygon and more than a half of the difference between the previous polygon and the area of the circle. By prop. 10.1, “by doing this continually,” the combined area of the circle segments thus formed becomes smaller than any number ε.[14]

The proof of prop. 12.2 continues by applying the so described exhaustion method. The proof is by contradiction. Suppose that $A_1 : A_2 \neq d_1^2 : d_2^2$ and for a circle of area $A \neq A_1$ it is true that $A : A_2 = d_1^2 : d_2^2$. If $A < A_1$ (Fig. 2), then by the just described exhaustion method it is possible to inscribe in the circle A_1 a polygon of area p_1 so that $A < p_1 < A_1$. If p_2 is the area of the similar polygon inscribed in the circle A_2, then, by prop. 12.1, $p_1 : p_2 = d_1^2 : d_2^2$, and because $A : A_2 = d_1^2 : d_2^2$, we also have $p_1 : p_2 = A : A_2$. However, because $p_2 < A_2$, then $p_1 < A$, which contradicts the fact that $A < p_1$. A similar contradiction arises for the supposition that $A > A_1$.

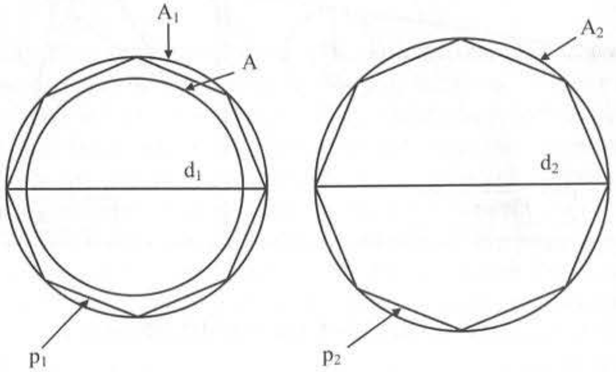


Figure 2

---

[14] This is also stated in prop. 1.6 of Archimedes’ *On the sphere and cylinder*: for a given circle it is possible, by inscribing polygons in it and then “by continually inscribing in the remaining segments,” to leave segments that are less than any given area.

A variant of prop. 12.2 is used by Archimedes in his *Measurement of the circle* with an implicit use of a spiral to accomplish the task.[15] In *On spirals*, prop. 18, Archimedes shows that if a spiral is drawn from a point O, and A is the end of the first turn of the spiral, then for the tangent AB to the spiral at the point A, the line OB perpendicular to the radius OA crosses the tangent at B and OB is equal to the circumference of the circle with the radius OA and the center at O (Fig. 3). Then, at the start of the *Measurement of the circle*, Archimedes shows that the area of any circle is equal to the right-angled triangle in which one side next to the right angle is equal to the radius, and the other side to the circumference; i.e., the triangle ABO in Fig. 3 has the area equal to that of the circle with radius OA. In his proof, Archimedes brings to contradiction two remaining possibilities: that the area of the circle is greater than the area of the triangle and that the latter is greater than the former. He shows it by producing, just like in Euclides' proof, a series of inscribed polygons so that for an *n*-sided polygon, a 2*n*-sided polygon is generated in the next step. The process is continued until the sum of areas of the circle segments between the circumference and the sides of the polygon are smaller than the difference between the areas of the circle and the triangle. But this, as Archimedes shows, leads to a contradiction. Similarly with the other possibility (the area of the circle is smaller than the area of the triangle) and series of circumscribed polygons. Significantly, the process of generating polygons is assumed to be possible regardless of the assumed difference between the two areas under scrutiny; hence, infinite divisibility is assumed in the proof.

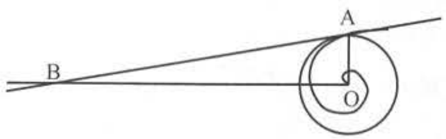


Figure 3

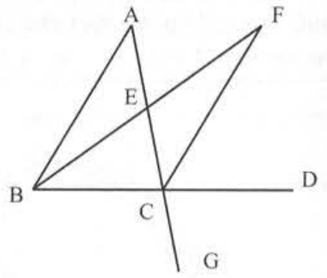


Figure 4

16.4 EUCLID AND ARCHIMEDES

Treating infinity in the way illustrated above permeates Euclid's *Elements* and the writings of Archimedes. Euclid systematizes the mathematics of his times, provides an excellent compendium that presents the accepted mathematical knowledge of ca. 300 BC. There may be very little original input from Euclid himself. Archimedes, on the other hand, is a first-rate mathematician presenting the cutting-edge research. For both of them, infinity is really a non-issue.

Infinity is assumed at the very outset of the *Elements*. Postulate 3 of book 1 states that it is possible "to draw a circle with any center and distance/radius" which allows for an infinity of different circles, assuming that there is an infinity of numbers (distances). Also, an infinite space is assumed which makes drawing circles of any size possible.[16] This is, in fact, clear from the proof of prop. 1.16 according to which, the angle ACD is greater than angles ABC and ACB of a triangle ABC (Fig. 4). To see it, consider the interval BF that bisects the side AC and for which BE = EF. Since AE = EC and BE = EF, then angle(AEB) = angle(FEC), and thus AB = FC and triangle(ABE) = triangle(CFE), hence angle(BAE) = angle(ECF). However, angle(ECF) < angle(ECD); therefore, angle(BAE) < angle(ACD). Similarly, starting from the bisection of BC it can be shown that angle(ABC) < angle(BCG) = angle(ACD). The proof holds in the Euclidean space, which is infinite, in particular the statement that the line CF is always inside the angle ACD; this is not universally true in the Riemannian (elliptic) geometry which allows for an unlimited space which is finite (the surface of the sphere).[17]

Infinity is also assumed more directly. Definition 23 in book 1 defines parallel lines as the ones which, being on the same plane and extended infinitely (εἰς ἄπειρον), do not cross one another (cf. also the famous postulate 5 in the same book that uses the analogous phrase ἐπ' ἄπειρον), which quite clearly assumes that the plane is infinitely extended in the Euclidean space. Prop. 1.12 states that it is possible to draw an interval perpendicular to an infinite line from a point not on that line. Clearly, the assumption of infinity is necessary here, otherwise the interval may never reach the line when drawn from some points (cf. Proclus, *In Eucl.* 284.4-16). Also, the proof of prop. 1.22 explicitly uses a line that is infinite in one direction.

Euclid takes for granted the existence of infinite sets. In the proof of prop. 7.31, a reference is made to an "infinite [series of] numbers." More cautiously, in definition 3 of book 10, he states that it can be proven that there exist "an infinite number of straight lines which are commensurable and incommensurable" with some line. Proofs for generating an infinity of an infinite number of lines incommensurable with some line is provided by propositions 10.10 and 10.115.[18]

Another important assumption made by Euclid is the continuity of magnitudes, which implies their infinite divisibility. The postulate 3 also assumes that circles of any indefinitely small size can be drawn, which assumes that the space is continuous.[19] This assumption becomes clearer in the proof of the very first proposition of book 1 which states that it is possible to construct an equilateral triangle ABC for any length AB. The proof begins by drawing circles with centers

---

[15] Cf. Thomas Heath, *A history of Greek mathematics*, Oxford: Clarendon Press 1921, v. 1, 230-231.

[16] Heath, *Thirteen books*, v. 1, 200.

[17] Cf. Heath, *Thirteen books*, v. 1, 200, 280.

[18] There are some reservations concerning authenticity of these two propositions, Heath, *Thirteen books*, v. 3, 32-33, 255.

[19] Heath, *Thirteen books*, v. 1, 199.

A and B and with radii AB (Fig. 5). From the point C of intersection of the triangles, two intervals are drawn to form the triangle ABC. The three sides of the triangle are obviously equal since they are all radii. In the proof, an assumption is made that the point C exists, that is, that the two circles do intersect which can only be assured if the circumferences are assumed to be continuous.[20] This assumption is made every time a construction requires generation of a point by an intersection of lines and circles.

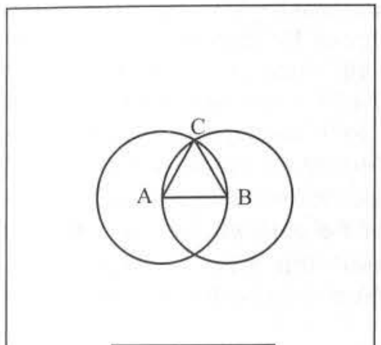


Figure 5

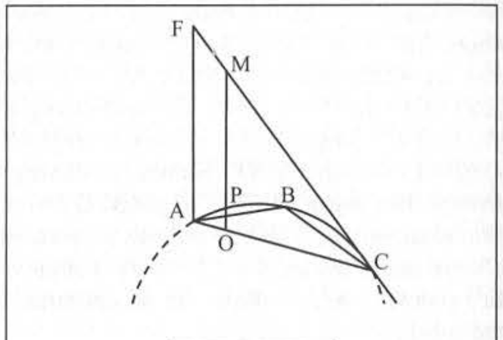


Figure 6

Continuity of magnitudes and thus infinite divisibility is also an underlying assumption made repeatedly by Archimedes. It is made when requiring in proofs of many propositions that continual bisection of intervals (e.g., *On the equilibrium of planes* 1.9 and 1.13), arcs (e.g., *On the sphere and cylinder* 9 and 10; *Measurement of the circle* 1), and angles (e.g., *On spirals* 23) can arrive at a particular size no matter how small the size can be. The assumption is made in particular when finding an area by a method that adumbrates integral calculus. Archimedes' mechanical argument is that he sees an area as the sum of an infinite number of parallel straight lines marked by the boundary of a particular figure and a volume as the sum of an infinite number of parallel plane sections marked by a particular solid. For example, consider prop. 1 of *The method treating of mechanical problems* which states that any segment ABC of a parabola is four-thirds of the triangle ABC with the same base and height as this segment (Fig. 6). When demonstrating this proposition, Archimedes states that "the triangle CFA is made out of all the parallel lines like MO and the segment CBA is made out of all the straight lines like PO," where the interval MO is parallel to the axis of the parabola and FC is the tangent at C. But the demonstration ends with the statement that this is not a real proof but some indication (ἔμφασίν τινα) of the validity of the proposition, or, as Archimedes remarks in the preface to the *Method*, his mechanical method is a way to make clear the meaning of some statement which should later

---

[20] Cf. Heath, *Thirteen books*, v. 1, 235, 242; Felix Klein, *Elementary mathematics from an advanced standpoint: geometry*, New York: Macmillan 1939, 197, 199; Howard W. Eves, Carroll V. Newsom, *An introduction to the foundations and fundamental concepts of mathematics*, New York: Holt, Reinhart & Winston 1965, 43-44.

be proven geometrically. In this way, infinitistic thinking appears to be more intuitive than a geometrical proof in which references to infinity are constrained.

It is interesting to see that a similar caution with respect to infinity is expressed much later by Nicomachus of Gerasa in his *Introduction to arithmetic*. Nicomachus says there that magnitudes and multitudes by nature are infinite – the former are characterized by infinity by division, the latter by infinity by addition. They can become the object of wisdom, not science, since science treats about limited things, i.e., about quantities carved out from multitude and sizes carved out from magnitude (*Intro.* 1.2.5). And yet, constantly in his book, Nicomachus uses the phrase "*ad infinitum*." There is no problem with using infinity intuitively, in relying on it as the base of analyses, proofs, and scientific investigations. It is just that infinity itself is not the province of science. Science can be precise, but it is not powerful enough to grapple with the overpowering problem of infinity itself. However, the problem cannot be dismissed altogether.

First, infinity constitutes an indispensable substrate for the finite. Whatever exists, exists due to the infinite background; whatever exists, is a manifestation of the infinite. As phrased by Proclus in his commentary on Euclid, in mathematics, understanding (διάνοια) grasps (λαμβάνει) the infinite not for the sake on the infinite, but for the sake of the finite (*In Eucl.* 285.24, 286.3-4), and this is an area with which science can grapple.

Second, the infinity is important for a nonmathematical reason. As Nicomachus states, happiness can be found solely through philosophy, i.e., in wisdom (1.2.3). Knowledge of infinity, then, is an avenue leading to a happy life. Happiness through the knowledge of infinity. We can recognize here a recipe presented centuries before Nicomachus by Epicurus. The idea does not have to be so abstract when infinity is seen in theological and eschatological context, as the desire of knowledge of divine infinity and infinite life in the hereafter, major concerns for Neopythagoreans like Nicomachus.

Because infinity is not a scientific concept, there is no major doubt cast on its usability in Greek mathematics. Infinity as such never became a subject of investigation by Greek mathematicians. It was always assumed to be obvious and basically unproblematic, but although it constituted a fundamental assumption of mathematical analyses, reflection on infinity was left to philosophers. Even the Pythagoreans, who occupied themselves with mathematical investigations, did not approach infinity from a mathematical standpoint, although, at the same time, they made infinity a foundation of their ontology. Also, the discovery of incommensurability and Zeno's paradoxes did not cast any major doubt on the validity of the use of infinity in mathematics.[21] They made mathematicians and philosophers see the problems that can arise when infinity is not used carefully and brought realization that its use should sometimes be restrained, but infinity always remained an indispensable foundation not only in mathematics but also in philosophical and theological reflection.

---

[21] Cf. Wilbur R. Knorr, Infinity and continuity: the interaction of mathematics and philosophy in Antiquity, in N. Kretzmann (ed.), *Infinity and continuity in Ancient and Medieval thought*, Ithaca: Cornell University Press 1982, 114, 121, 135, 143.